June 2015

Formal Theory Exam

Dept of Political Science, Yale University

Please do all three questions and show your work.

Problem 1. There is a polity where voters of income w have utility functions over the tax rate t given by $v(t;w)$. There are two parties, the Left and the Right. The Left party consists of two factions: the Opportunists wish to maximize the probability of victory, and the Militants wish to maximize the utility of a voter whose income is $w_L < \mu$. That is, the payoff function of the Left Militants is:

$$\Pi^{LM}(t_L,t_R) = v(t_L;w_L) .$$

The payoff function of the Left Opportunists is:

$$\Pi^{L}(t_L,t_R) = \pi(t_L,t_R)$$

where $\pi(t_L,t_R)$ is the probability that t_L defeats t_R .

Likewise, there are two factions in the Right party. The Right Opportunists wish to maximize the probability that Right wins; the Right Militants wish to maximize $v(t;w_R)$, for some ideal constituent $w_R > \mu$.

It is important to realize that the Militant factions do not care about winning as such. They care about *sticking to principles* in the sense of proposing policies that their constituency likes.

A. Consider the following Nash bargaining problem, which takes place in the *L* party. Suppose the *R* party has proposed the policy t_R. The Opportunists and Militants in Left *bargain* with each other concerning what policy to propose as t_L . The threat point is derived as follows. If the *L* factions cannot come to an agreement on what policy to propose, then party *R* wins the election by default: that is, with probability one. And the constituents of the Left must live with the policy t_R . Thus the threat utility for the Left Opportunists is zero and the threat utility for the Left Militants is $v(t_R;w_L)$.

Write down the Nash bargaining problem for the Left factions.

B. In like manner write down the bargaining problem for the Right factions, when facing a Left policy proposal of t_L .

C. Now *propose a concept* of political equilibrium in this setting. Does this equilibrium look familiar to you?

D. Suppose in the polity the distribution of earnings, w, is given by a cdf F with mean μ . The utility function of a typical citizen is $u(x,G) = x + G - \frac{1}{2\mu}G^2$ where x is private income and G is the value of a public good, produced from tax revenue. The value of *G* is measured in its per capita cost. Thus, the indirect utility function of a voter with income *w* is $v(t;w) = (1-t)w + t\mu - \frac{1}{2\mu}(t\mu)^2$.

Let $w_L < \mu < w_R$, and suppose the probability function is given by the *error-distribution model*. Compute the probability function $\pi(t_L, t_R)$.

E. Now write down the first-order conditions for the equilibrium you defined in part C, for an equilibrium where $0 < \pi(t_L, t_R) < 1$.

F. From the first-order conditions, deduce an upper bound on the policy of the Left party and a lower bound on the policy of the Right party. Can you give some intuition for these bounds?

G. Thus far, we have taken the 'constituents' of the Left and Right parties to be single voters, with incomes w_L and w_R . Can you now propose an equilibrium concept which *endogenizes* what the party constituencies will be? An equilibrium will now consist of a pair of policies (t_L, t_R) and a pair of constituencies *L* and *R* (each a set of voters) such that $L \cup R$ is the entire polity.

H. If you have succeeded in endogenizing the party constituencies, show that both equilibrium policies must be weakly Pareto efficient .

Problem 2.

A. State and prove Nash's bargaining theorem.

B. Suppose there are two individuals, who jointly own a piece of land. One of them is incapable of working. The other can produce wheat from the land according to the production function $F(L) = W = aL$ where *W* is wheat in bushels and *L* is labor time. The vNM utility functions of these individuals are:

$$u(W) = \sqrt{W}, \quad v(W, L) = \sqrt{W(1-L)} ,$$

where *v* is the able-bodied person. The land cannot be used until the two owners agree on how much wheat will be produced and how it will be shared, and neither has any outside option. Suppose that, in the absence of an agreement, there is no production. Solve this as a Nash bargaining problem.

C. What does this say about incentive compatibility of Nash bargaining?

D. *Now* let us suppose that the skilled individual has an outside option which is worth *k* to him in utility terms. (That is, if bargaining fails, he can achieve utility *k* using the outside option.) Compute the value of *k* such that at the Nash bargaining solution the disabled individual gets nothing. Is the land used or not?

Problem 3.

Consider a political economy with two groups, a government (G) and a rebel group (R). The two groups negotiate over the division of the spoils of office in the shadow of a civil war. Time is discrete and the game is played an infinite number of periods. All agents are risk neutral and discount the future by discount factor $\delta<1$.

In every period, the government offers a division of the spoils of office of period *t*, where the rebel group gets x_t, or decides to fight the rebel group; if the government chooses not to fight the rebel group, the latter decides whether to accept or reject the government's office. If the rebel group accepts the offer, it is implemented. If it rejects the offer, the two groups fight over the spoils of office.

The initial value of the spoils of office is normalized to one per period. If the two groups fight each other, then the rebel group wins with probability p_t. The winner of the conflict gets the current spoils of office. In addition, the winner consolidates power with probability γ. If the winner consolidates power, then the game ends and the winner has access to additional contingent spoils $S\geq0$ in every future period. If the winner does not consolidate power, then the game continues with the above timing, with the government offering a division of the spoils of office. Fighting is costly. Each player pays *c* in any period in which there is a fight.

We want to know when peace can be sustained in this game.

A)Assume that the balance of power is constant over time, i.e. $p_t = p$ for all t, where $p>c$.

Is there a unique Markov Perfect Equilibrium (MPE) of this game? Is there an MPE where peace prevails? Is there an MPE of this game where war happens along the equilibrium path?

B)Assume that the balance of power changes over time, i.e. in every period, unless a group has consolidated power, then p_t is either p, as described above, or 0 (a p_t equal to 0 should be interpreted as a temporary weakness for the rebel group; it does not mean that the government has consolidated power). Assume that p_t take a value of p with probability θ.

Is there a unique Markov Perfect Equilibrium (MPE) of this game? Is there an MPE where peace prevails? Is there an MPE of this game where war happens along the equilibrium path?

C)Assume that the balance of power depends on the division of the spoils of office, and the government consolidates power if the rebel group accepts a sufficiently small share of these spoils. In particular, assume that the balance of power begins at $p_1 = p$; there is a value $x_{\min}$ such that if $x_t > x_{\min}$, then $p_{t+1} = p$, and if $x_t \leq x_{\min}$, then the game ends after period t, the government consolidates power and has access to the additional contingent spoils S in every future period.

Is there a unique Markov Perfect Equilibrium (MPE) of this game? Is there an MPE where peace prevails? Is there an MPE of this game where war happens along the equilibrium path?

D)Evaluate the lessons of the model. Is it easier/harder to sustain peace when the probability that a group consolidates power γ is high? Is it easier/harder to sustain peace when the value of the contingent spoils S is high?