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Formal Theory Exam
Department of Political Science
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Please answer all three questions.

Question 1

A. State the Hotelling-Downs median voter theorem and present a proof.

B. Suppose there is a polity whose voters all have Euclidean preferences on the policy space $\mathbb{R}^2$. A voter's *type* is his ideal point; thus, a voter of type (a,b) have preferences represented by the utility function

$$v(x,y;a,b) = -(x-a)^2 - (y-b)^2 .$$

There is a distribution **F** of voter types on the type space $\mathbb{R}^2$.

(i) By drawing pictures, derive a condition on **F** which is necessary for the existence of a Condorcet winner in the voting game. State the condition precisely.

(ii) Conclude that the existence of a Downsian equilibrium in this model is a singular occurrence.

C. Consider the political economy where there is a fixed distribution of income, **G**, on $\mathbb{R}^2_+$, whose mean is μ. A voter's income, *y,* is her type. The policy space is $T = [0,1]$. The utility function of a voter of type y is given by $v(t;y) = (1-t)y + t\mu$.

Suppose that there is some uncertainty in elections. Given two policies (t_1,t_2), we represent the uncertainty with the *error distribution model*: that is, the fraction of the vote for t_1 will be the predicted vote, $\pm\beta$.

(i) Compute the Hotelling-Downs equilibrium for this model.

(ii) Define Endogenous Party Nash-Wittman equilibrium (EPNWE), and compute the EPNWE equilibrium in this model.

(iii) Explain why the computation of the EPNWE equilibrium is particularly simple in this model.

Question 2

Consider the following model. A decision maker (d) has access to three different advisors, advisor 1, advisor 2, and advisor 3. The decision maker must select a policy $p \in [0,1]$. The decision maker's preference over policies depends on the state of the world $\omega \in [0,1]$, and is represented by her utility function, $u_d : [0,1] \times [0,1] \to \mathbb{R}$, where

$$u_d(p,\omega) = -(p-\omega)^2$$

for each $p \in [0,1]$ and each $\omega \in [0,1]$.

The utility function of each advisor $j \in \{1,2,3\}$ is given by

$$u_j(p,\omega) = -(p-\beta_j-\omega)^2,$$

where $\beta_j \neq 0$. Hence, for any state of the world ω, advisor j's most preferred policy is $\beta_j + \omega$.

The game begins with Nature drawing the state of the world from the interval $[0,1]$ according to the uniform distribution. The state of the world is observed by each of the advisors. However, it is not observed by the decision maker. Upon observing the state of the world, each advisor j sends a costless report $r_j \in [0,1]$ to the decision maker. The advisors send their reports simultaneously. After observing all advisors' reports, the decision maker chooses which policy to implement. All descriptions of the game in the above are common knowledge.

A. Prove that there exists a perfect Bayesian equilibrium where every advisor truthfully reports the state of the world for every possible state of the world.

B. Now suppose the decision maker has only one advisor, say advisor 1.

(i) Prove that there does not exist a perfect Bayesian equilibrium in which advisor 1 truthfully reports the state of the world for every possible state of the world.

(ii) Find a perfect Bayesian equilibrium where, regardless of the report sent by the advisor, the decision maker chooses policy $\frac{1}{2}$. (You need to fully specify equilibrium strategies and beliefs.)

(iii) Suppose $\beta_1 \in (0, \frac{1}{4})$. Find a perfect Bayesian equilibrium in which there exists $\omega^* \in (0,1)$ such that the policy outcome is $\frac{\omega^*}{2}$ whenever the state of the world is less than or equal to ω^*, and it is $\frac{\omega^*+1}{2}$ whenever the state of the world is greater than ω^*. (You need to compute the exact value of ω^* and fully specify equilibrium strategies and beliefs.)

Question 3

Consider a polity with two groups, A and B, which divide the spoils of office. There are two possible political regimes, democracy (D) and non-democracy (ND). The political regime determines the allocation of de jure political power which, in turn, affects the division of the spoils of office. For simplicity, we assume that in any regime, there are periods where de jure power is contested and others where it remains in the hands of the incumbent. If de jure power is contested in period t, it is won by group A with probability p_D in democracy and p_{ND} in non-democracy (and won by group B with probability $1 - p_D$ and $1 - p_{ND}$ in democracy and non-democracy, respectively). After winning de jure power in period t, a group maintains de jure power for a number of periods, n_D in democracy and n_{ND} in non-democracy. Each group discounts future payoffs with the discount factor β.

We assume that the process of obtaining and securing de jure power is a function of the regime type. More specifically, de jure power is obtained through elections in democracy and through violence (or the threat of violence) in non-democracy. Groups may differ in their ability to win elections and their ability to generate violence. Without loss of generality, assume that group A is better in producing violence than electoral success, $p_D < p_{ND}$. Moreover, assume that it is more difficult for a group without de jure power to replace an incumbent in non-democracy, because of collective action problems in using violence ($n_D < n_{ND}$). Finally, the use of violence is inefficient, so that the spoils of office are equal to 1 in any period under democracy and $\theta < 1$ in non-democracy. We want to characterize the conditions for a democratic transition, assuming that democracy is an absorbing state.

Let the timing of the game be as follows. In a period where power is contested, the timing is as follows:

1. Nature determines which group has de jure power

2.ND In non-democracy, the group with de jure power picks the regime type (D or ND) and implements a division of the spoils of office.

2.D In democracy, the group with de jure power implements a division of the spoils of office.

In a period where power is not contested, the group with de jure power implements a division of the spoils of office.

1. (5 pts) Define a Markov Perfect Equilibrium (MPE) of this game.

2. (10 pts) Solve for the MPEs of this game. Does the condition for democracy become more or less stringent as we change the parameters of the model (p_D, p_{ND}, n_D, n_{ND}, θ)?

3. (15 pts) Now assume that if the group with de jure power maintains the non-democracy, the proposed division of the spoils of office must be approved by the group out of power before it is implemented. If it is approved, it is implemented. If it is rejected, then we repeat the stage game, without discounting, until either the offer is accepted or the country becomes democratic (in other words, Nature re-draws the identity of the group with de jure power; this group picks the regime type and offers a division of the spoils of office; if the country remains non-democratic, the group without de jure power decides to accept or reject it).

Define and solve for the MPEs of this game. Does the condition for democracy become more or less stringent as we change the parameters of the model (p_D, p_{ND}, n_D, n_{ND}, θ)?

4.(20 pts) Discuss your results with respect to 2 of the following 3 theories of democratization: Lizzeri and Persico (2004), Acemoglu and Robinson (2006), Debs (2010), first describing the main result of the references you pick.