August 2007

Formal Theory Exam

Please answer all questions.

Political Equilibrium

A. Define a *political environment* and :

(i) Define Hotelling-Downs equilibrium

(ii) State a standard theorem concerning existence of H-D equilibrium

(iii) Give a condition which will guarantee existence if the policy space is two dimensional

(iv) Discuss some pros and cons of H-D equilibrium.

B. Define Nash-Wittman equilibrium. Discuss some ways in which N-W equilibrium is superior to H-D equilibrium.

C. (i) Define *party unanimity Nash equilibrium* (PUNE) for two parties where there are Opportunist, Reformist, and Militant factions.
(ii) Define an 'Opp-Mil' equilibrium as a PUNE-like equilibrium where there are only Opportunist and Militant factions.

Prove that (under some reasonable conditions) the set of PUNEs coincides with the set of 'Opp-Mil' equilibria.

(iii) In the case of differentiability of the relevant functions, state necessary conditions for a pair of policies, which are both interior in the policy space, to be a PUNE.

(iv) Suppose that there are *three* political parties. Write down the necessary conditions for an interior PUNE. What do you think is the dimensionality of the manifold of PUNEs in this case?

(v) Suppose there are p parties and f independent factions in each party. What do you think is the dimensionality of the manifold of PUNEs?

(vi) Return to the standard case where p=2=f. Can you give an interpretation for the non-uniqueness of PUNE in terms of 'missing data?'

Game Theory and Politics

The policy space is $X = [0, 1]$. There are two political parties D and R, each endowed with its position in X. Let x^D and x^R denote the respective positions of the parties. We assume $x^D < x^R$. There are two policy-making institutions: the President and the Congress. The policy outcome is determined by two factors: which party holds the presidency and which party holds a majority of the Congress. Specifically, if $I \in \{D, R\}$ holds the presidency and $J \in \{D, R\}$ holds the Congressional majority, then the policy outcome is

$$\alpha x^I + (1 - \alpha)x^J,$$

where α is an exogenous parameter measuring the institutional power of the President relative to the Congress. We assume $\frac{1}{2} < \alpha < 1$.

There are three electoral districts. There is a voter in each district. Let $N = \{1, 2, 3\}$ denote the set of districts or voters. Each voter $i \in N$ is endowed with his/her ideal policy $t_i \in X$ and a utility function $u_i : X \to \mathbb{R}$ such that, for every $x \in X$,

$$u_i(x) = -(x - t_i)^2.$$

We consider a strategic form game among the voters. In it, the presidential election and the congressional election are simultaneously held. So, each voter casts two ballots: D or R in the presidential election and D or R in the congressional election. Abstention is not allowed in either of the elections. All votes are cast simultaneously. We may write a player's action space as $A = \{DD, DR, RD, RR\}$, where IJ means "vote for I in the presidential election and vote for J in the congressional election." We say an action $IJ \in A$ is a straight-ticket vote if $I = J$ and a split-ticket vote if $I \neq J$.

The president is elected by a nationwide plurality rule, whereas a voter is able to dictate the partisanship of the member of Congress that represents his/her district. After the elections, voters receive the payoff from the policy outcome that is the aforementioned weighted average of the position of the presidential party and the position of the party holding at least two seats in the Congress. For convenience, let x^{DR} denote the policy outcome when D holds the presidency and R holds the congressional majority, and define x^{RD} in a similar way. Throughout we assume that voter preferences, as well as the game form, are common knowledge.

1. Show that if voter i's ideal point t_i is less than or equal to $\frac{x^D+x^{DR}}{2}$, then voter i has a weakly dominant strategy.

2. What is the set of all voter types (ideal points) that have a weakly dominated strategy but no weakly dominant strategy?

3. Assume that $t_1 < x^D$, $\frac{x^D+x^{DR}}{2} < t_2 < \frac{x^D+x^R}{2}$, and $t_3 > x^R$. Find all pure strategy weakly undominated Nash equilibria – i.e., Nash equilibria in which no player plays a weakly dominated strategy.

4. Assume that, for every $i \in N$, $\frac{x^D+x^{DR}}{2} < t_i < \frac{x^D+x^{RD}}{2}$. Show that there is a pure strategy weakly undominated Nash equilibrium in which every voter casts a straight-ticket vote. Show that this equilibrium does not survive IEWDS.

Intertemporal commitment in a Markov game

Several recent contributions in formal political economy emphasize that the difficulty of committing to policies over time can be an important source of political conflict. This question asks you to use formal tools to study this issue.

Consider the following infinite-horizon version of a divide-the-dollar game. In each period, player 1 offers $z_t \in [0, 1]$ to player 2, who can either (i) accept the offer or (ii) refuse the offer and quit the game forever. Both players discount the future at the common rate $\beta \in (0, 1)$. Each period of the game is characterized by a state, $s_t \in \{L, H\}$, for "Low" and "High," respectively. If player 2 ends the game in a period in which the state is low, both players receive a payoff of zero forever. If player 2 ends the game in a high state, players 1 and 2 receive a constant stream of strictly positive per-period payoffs of θ_1 and θ_2, respectively, where

$$\theta_1 + \theta_2 < 1 \tag{1}$$

The states fluctuate over time, following a stationary Markov process with the following transition probabilities:

$$\begin{pmatrix} 1-q & q \\ p & 1-p \end{pmatrix}$$

Thus, starting from state L, the probability of remaining in state L in the next period is $(1 - q) \in (0, 1)$ and the probability of transitioning to state H is q; starting in state H, the probability of remaining in state H in the next period is $p \in (0, 1)$ and the probability of transitioning to state L is $(1 - p)$.

1. First, define formally what is meant by a subgame-perfect Nash equilibrium (SPE) and a Markov-perfect equilibrium (MPE) of this game. What is the relationship between the subgame-perfect and Markov-perfect equilibrium concepts?

2. Prove by contradiction that in any MPE, $z_t = 0$ in any period in which the state is low.

3. Show that the maximum value that can be given to player 2 in any MPE, starting in state H, is

$$V_2^H = (\frac{1}{1-\beta})(\frac{1-\beta(1-q)}{1-\beta(p-q)}) \tag{2}$$

where the notation V_2^H gives the value function of player 2 in state H.

4. Show that there may exist values of θ_2 that satisfy (1) such that player 2 quits the game when the state $s = H$ is reached. Write down an inequality that characterizes these values of θ_2.

5. Note that the outcome in the previous step is "inefficient" in the sense that if the players could write a binding contract, they could find a division of the flow of dollars that would leave both players better off than they are if player 2 ends the game. Show this.

6. Now, show that the incidence of the inefficient outcome increases in β. (Though you are not asked to show this here, this result persists when we consider SPEs). Contrast your finding to standard results in the theory of repeated games. What accounts for this result?

7. Can you think of any political situations in which a greater "shadow of the future" might drive these kinds of inefficient outcomes?