Question for PSC 518 (Introduction to Game Theory)

Consider a two-player ultimatum (also known as take it or leave it) bargaining game where either a peaceful settlement or war can occur. Country 1 makes an offer to split a territory that is normalized to size 1 and then country 2 decides to accept or reject the offer. Denote 1's offer by x with the interpretation that if the offer x is accepted then 1's payoff is x and 2's payoff is 1-x. If 2 rejects the offer then a war occurs. Let the payoff to country 1 from war be given by p-c and country 2's payoff from war be given by 1-p-k. The term p is interpreted as the probability that 1 will win the war (so it is between 0 and 1). Further assume that while both players know the exact value of c, the value of k is only known by country 2. Assume that country 1 believes the value of k is drawn from a distribution function f on the set [0,1].

a. What is the appropriate equilibrium condition for analyzing this game?
b. First assume that f is the uniform distribution and derive the equilibrium to this game.
c. Derive the equilibrium offer and the equilibrium probability of war as a function of the parameters c and p.
d. Under what conditions does the equilibrium involve a positive probability of war?
e. Under what conditions (if any) is the equilibrium probability of war increasing in the parameter p? Under what conditions (if any) is the equilibrium probability of war deceasing in the parameter p.
f. Now generalize and assume that the distribution f is not necessarily the uniform but rather it is a continuous and differentiable CDF on the unit interval. Provide an implicit characterization of the equilibrium?
g. Under what conditions if any is the equilibrium probability of war increasing in the parameter p? Under what conditions if any is the equilibrium probability of war decreasing in p?
h. Comment on what this exercise tells you about how military capacity impacts the odds of war.

Question for PLSC 519, Formal Models of Domestic Politics

1. **Observable and Unobservable Effort in the Two-Period Career-Concerns Model.**[1] This is a modified version of the career-concerns model of political agency (see Section 7.2.1 of the Gehlbach textbook). Consider a two-period model with the following actors: an incumbent politician, a challenger, and a population of identical voters. All actors share a common discount factor of $\delta \in (0, 1)$.

   The two periods consist of an election period ($t = 1$) and a post-election period ($t = 2$). In each period, the officeholder chooses how much effort to exert of two types: effort that is perfectly observable to voters, $o_t \in [0, \infty)$, and effort that is unobservable/hidden to voters, $h_t \in [0, \infty)$. In each period, the officeholder earns an exogenous wage, w, and pays a total effort cost of $\frac{1}{2}(o_t + h_t)^2$.

   Voters receive utility each period of $u_t = \theta + \beta o_t + (1 - \beta)h_t$, where θ represents the competence of the politician in office, o_t represents the officeholder's chosen level of observable effort, h_t represents the chosen level of hidden (unobservable) effort, and $\beta \in (0, 1)$ is a parameter of the model. As before, θ is a random variable drawn from a differentiable distribution function, F, with density f and $E(\theta) = 0$. Assume that the support of f is the real number line.

   Competence θ is drawn the first period that a politician is in office after the officeholder makes his initial effort choice. Once drawn, competence θ is observable to the officeholder only and carries over period-to-period. At the end of period 1, voters determine whether to retain the incumbent for period 2. If they choose not to retain the officeholder, a challenger is elected with $E(\theta) = 0$. To summarize, the timing is:

   - ($t = 1$): The incumbent chooses observable effort o_1 and hidden effort h_1
   - ($t = 1$): The incumbent's competence is revealed. Voters receive utility u_1 and observe observable effort o_1. (They cannot observe politician competence or hidden effort.)
   - ($t = 1$): Voters choose whether to retain the incumbent
   - ($t = 2$): The election winner chooses o_2 and h_2
   - ($t = 2$): If the challenger wins the election, her competence is revealed. Voters receive u_2 and observe o_2.

[1] This problem is slightly adapted from the additional exercises of the Gehlbach textbook.

Analyze this model as follows:

(a) Derive the equilibrium level of observable and hidden effort in period 2. Use this to derive the condition for voters to retain the incumbent.

(b) Show how voters impute the competence of the incumbent based on their observed utility u_1, the observed effort of the incumbent o_1, and their beliefs about hidden effort h_1. (Hint: write down the expression for u_1 as a function of θ, o_1, and h_1.) Use this to derive the probability that the incumbent is reelected.

(c) Derive the incumbent's equilibrium choice of observable and hidden effort in period 1.

(d) How does the total effort in period 1, $e_1 = \beta o_t + (1 - \beta) h_t$, depend on the relative importance of observable and unobservable effort, as captured by the parameter β? Provide a brief (i.e., no more than a paragraph) substantive interpretation.