Inequality under Democracy: Explaining the Left Decade in Latin America∗

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ABSTRACT

Inequality is generally thought to affect the electoral fortunes of the left, yet the theory and evidence on the question are unclear. This is the case even in Latin America, a region marked by enormous inequalities and by the stunning return of the left over the last decade. We address this shortcoming. Our game-theoretic model reveals that the probability that the left candidate is elected follows an inverted U-shaped relationship. At low levels of inequality, the rich do not bribe any voters and poor voters are increasingly likely to vote for the left candidate based on redistributive concerns. At high levels of inequality, the rich want to avoid redistribution and bribe poor voters, causing the left candidate to be elected with decreasing probability. We find support for our hypothesis, using 110 elections in 18 Latin American countries from 1978 to 2008.

The combination of inequality and democracy tends to cause a movement to the Left everywhere. This was true in Western Europe from the end of the century until after World War II; it is true today in Latin America. The impoverished masses vote for the type of policies that, they hope, will make them less poor.

—Jorge Castañeda 2006

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High inequalities bias the political rules of the game and mold polities in favor of the wealthy and privileged. [T]hey do so (to different degrees) whether regimes are authoritarian or democratic.

—Terry Karl 2003

INTRODUCTION

Over the last decade, the left in Latin America has been on a roll. Starting with Chávez’s breakthrough victory in Venezuela in 1998, leftist governments quickly surged to power in the largest three countries in the region: Chile (2000), Brazil (2002), and Argentina (2003). Despite Latin America’s well-deserved reputation for electoral volatility (e.g. see Roberts and Wibbels, 1999), in each of these countries leftist governments were also re-elected with overwhelming support. Meanwhile, the left has continued to chalk up a string of impressive victories throughout the rest of the region, including: Uruguay (2004), Bolivia (2005), Ecuador (2006), Nicaragua (2006), and Paraguay (2008). Even in the handful of countries where leftist parties have not yet managed to win power, the left has also been on the rise. In Mexico, the PRD’s presidential candidate, López Obrador lost the 2006 elections by a hair. In the same year, Peru’s left-wing candidate, Ollanta Humala, came out ahead the first round, losing in the second round by just a little more than 5% of the vote to the center-left APRA candidate, Alan García. In Colombia, arguably the right’s biggest stronghold in Latin America, the new leftist party, Alternative Democratic Pole, did unexpectedly well in the 2006 elections (Castañeda and Nava, 2007).

Not surprisingly, Latin America’s sudden swerve to the left has generated a considerable amount of interest.¹ Whereas much of the early academic debates centered primarily on conceptual issues — are there one, two, or multiple lefts in Latin America? — scholars also quickly set about trying to explain the left’s stunning electoral success. A central claim of the emerging literature has been that the rise of the left in Latin America is somehow linked to the failure of the so-called Washington Consensus. Neo-liberal reforms may have helped to end bouts of hyper-inflation in the early 1990s, but because these policies ultimately failed to generate sustainable growth, alleviate poverty, and ameliorate the vast inequalities that mark much of the region, the left is now reaping the benefits. At the same time, however, there has been a noticeable reluctance to attribute too much causal weight for the left’s recent success to underlying structural economic conditions, particularly inequality. After all, if Latin America has long been the most unequal region in the world, why is the left, which is essentially defined by its commitment to redistribution, only now being elected?

¹ Both Foreign Affairs (May/June 2006 Volume 85) and the Journal of Democracy (October 2006 Volume 17, No. 4) published a series of articles related to the rise of the left. Also in 2006 the Latin American Program at the Woodrow Wilson Center at Princeton launched a three-year project entitled, “The ‘New Left’ and Democratic Governance in Latin America.” In 2008, Harvard University sponsored a conference on the same theme, entitled, “Latin America’s Left Turn: Causes and Consequences.”
Yet, as we argue in this paper, posing the question in this way is problematic for at least two major reasons. First, from an empirical point of view, we concur that inequality is, and has long been, shockingly high throughout Latin America. But, we also contend that because the distribution of wealth varies both across countries and over time it is premature to dismiss inequality as part of the causal story. Second, from a theoretical point of view, we are intrigued by the apparent inconsistencies that mark discussions about just how inequality shapes political outcomes under democracy. Consider the two quotes cited above. If Castañeda is right, inequality in Latin America and elsewhere should simply drive poor voters who are in the majority to elect politicians that will redistribute resources. If Karl is correct, however, the situation is bleaker. Harkening back to Michel’s Iron Law of Oligarchy, Karl’s intuition is that the more the haves have the more resistance they will mount to keep the have-nots from gaining power; hence the cycle of inequality continues, regardless of regime type.

In this paper, we contend that both views tap into something essential about inequality under democracy, but neither is wholly satisfying. To explain why, we develop a game-theoretic model that shows how under competitive democracy the incentives of the poor to support the left and the rich to block it depend on the level of inequality. Our game contains a finite number of voters, divided into two groups: the rich (or wealthy) and the poor. Two candidates, called the right and the left candidates respectively, compete for office. They pick a single policy, a tax rate, which is applied proportionally on income. Tax proceeds, minus any inefficiency associated with collecting taxes, are redistributed lump-sum to all citizens. Candidates are purely ideological and offer the platform favored by their constituency (the rich and the poor, respectively). Candidates also differ on other dimensions, which we summarize by a valence shock incurred by all citizens. We also assume that there is a single lobby which offer bribes to get the candidate on the right elected. This assumption follows from one of the following two postulates. First, the rich are a smaller group, have more resources, and should be better able to solve their collective action problem (Olson, 1971). Second, because incumbents tend to have greater credibility in the disbursement of clientelistic goods, whenever the party on the right is in control of the government, the assumption is also satisfied.2

Then we show that the probability that the left candidate is elected follows an inverted U-shaped relationship. At relatively low levels of inequality, redistribution is low enough that it is not optimal for the rich lobby to offer any bribes. Since redistribution increases with inequality, the median voter (who is poor) is increasingly willing to vote for the left candidate. When inequality is sufficiently high, the rich lobby bribes a minimum winning coalition of voters. As inequality increases, the lobby’s willingness to get the candidate on the right elected increases, since it fears greater and greater redistribution. Of course, the poor’s willingness to elect the left candidate also increases with inequality. Yet we show that the net effect unambiguously favors the rich. Since taxation is inefficient, an increase in the tax rate is worth less to the poor than to the rich. Therefore, as inequality increases, the rich’s incentive to avoid the election of the left dominates and they consequently offer enough bribes to decrease the probability that the left candidate is elected. Applying

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2 For a longer discussion in the context of Latin America, see the Empirical analysis section.
this framework to Latin America, we argue that the left’s success can be explained by a relative convergence of inequality in the region.

In focusing on the level of inequality as our main causal variable, our theory contributes to a well established literature in comparative politics (Acemoglu and Robinson, 2000, 2001, 2006; Alesina and Perotti, 1996; Boix, 2003; Muller and Seligson, 1987). But, whereas the bulk of this work concentrates on the effects of inequality on democratization and regime change, we join in the emerging effort to explore the effects of inequality once the rules of democracy are already in place (Ziblatt, 2009). Using a political economy approach, the twist to our theory lies in explaining precisely how the level of inequality continues to affect political outcomes in environments such as present-day Latin America, where the rules of the democratic game are, on the whole, accepted.

The model closest to ours is Acemoglu and Robinson (2008), who show how elites can influence political outcomes even if they do not have the legal (or de jure) advantage. They compare the elite’s rate of success in democracy and non-democracy and study, among other things, the way that this rate of success varies with the size of the stakes for determining economic policy (which we could translate, in our framework, as inequality). They conclude that the probability that a pro-poor policy is implemented in a democracy is strictly decreasing with the size of stakes for determining economic policy. By contrast, we predict that this probability should first be increasing and then decreasing with inequality. The difference comes from our treatment of the process for determining economic policy. In Acemoglu and Robinson (2008), an individual can increase the probability that his favorite outcome is implemented only by investing in de facto power (through practices such as intimidation, lobbying and vote-buying). Given that the poor face an insurmountable collective action problem, they do not invest in de facto power. Only the rich invest in de facto power and, naturally, as the size of the stakes increases, they invest more in de facto power and the likelihood that the pro-poor policy is implemented goes down. In our model, an individual can influence the electoral outcome by exercising his de jure power (i.e., by voting). As inequality increases, the poor are more willing to vote for redistribution. For low enough levels of inequality, the benefit of exercising de facto power for the rich is too low, compared to its cost, so that the poor’s increasing willingness to vote for the left party is solely operating and translates into a higher likelihood that the left party is elected. Only when the rich do invest in de facto power (through vote-buying) does their increased resistance to redistribution determine the electoral outcome.

More generally, our model provides a new angle to understanding the relationship between inequality and redistribution. Meltzer and Richard (1981) argue that the expansion of the franchise explains the rise of the Welfare State in the West: as the median voter gets poorer, then the political equilibrium favors greater redistribution. Subsequent evidence on the matter has been inconclusive at best (see in particular Benabou, 1996 and

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3 While we offer a general model of vote-buying, we believe that it is particularly appropriate to explain the rise of the left in Latin America, with its combination of democracy, relatively weak rule of law (so that vote-buying is possible), moderate levels of income (as will be shown in Proposition 1) and incumbency advantage of right parties.
Perotti, 1996 and the discussion in Persson and Tabellini, 2000, 52, pp. 121–123). In a recent article, Iversen and Soskice (2006) go a long way toward resolving the specific question of why leftist parties get elected in advanced industrial democracies, arguing that PR systems are more favorable for the left than majoritarian systems. But this argument has limited reach in explaining variation in a region such as Latin America, where all countries have presidentialist systems. Another common feature of politics in Latin America is the relatively weak rule of law, which makes such countries vulnerable to vote-buying. We exploit this feature in our explanation of the rise of the left in the last decade.

On that note, our model also contributes to a rich literature on vote-buying. We introduce a few distinctive features: there is aggregate uncertainty about the preferences of the electorate and the lobby represents a given group of voters, maximizing their ex ante utility. In this set-up, the lobby faces a risk-return trade-off in setting its optimal bribes. We can then answer an empirical question from the clientelism literature: when (and how strongly) does a lobby target poor voters, as a function of the stakes of the election (in this case, inequality)?

The remainder of the paper unfolds as follows. The next section presents our formal model. We test our model against several alternative explanations using an original cross-national time-series dataset on elections in 18 Latin American democracies between 1978 and 2008. In addition to finding strong support for our main hypothesis, we bring fresh evidence to bear on a series of alternative hypotheses rooted in theories ranging from retrospective voting and democratic consolidation to post-Cold War politics and the legacy of mass-mobilizing party systems. The Conclusion section summarizes our results and discusses several implications that emerge from our analysis. Proofs are relegated to the appendix.

MODEL

There is a set $N$ of voters, who belong to one of two groups, the rich or wealthy ($W$) and the poor ($P$). Write $g(i)$ as the group of individual $i$ ($g(i) \in \{W, P\}$). The poor constitute

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4 Other solutions to the Meltzer and Richard (1981) puzzle include Campante (2007). In his model, ideological parties pick a platform, compete for campaign contributions from voters and the policy implemented is a weighted average of the policy platforms, with weights given by the vote share of each party. He shows that at high levels of inequality, redistribution decreases because parties propose platforms preferred by wealthier individuals, who can offer greater contributions. Interestingly, the model predicts an inverted U-shaped relationship between inequality and redistribution.

5 The first paper modeling vote-buying (by a lobby) may be Snyder (1991). Seminal papers include Groseclose and Snyder (1996) and Banks (2000), studying the optimal size of a winning coalition when two groups can bribe voters. A few papers have approached political campaign promises in the framework of a Colonel Blotto game. Myerson (1993) studies a game where two parties spend a fixed budget in bribing voters for their support, assuming that the budget has to hold in expectation only. More recent contributions in the vote-buying literature include Dal Bo (2007) and Dekel et al. (2008).
the larger group in the population. Let $|S|$ be the size of set of individuals $S$. We have $|P| > |W|$ and $|N|$ odd.

The income of individual $i$ is $y^\ell(i)$, where

\begin{align*}
y^W = \frac{\theta}{|W|} y \\
y^P = \frac{(1 - \theta)}{|P|} y
\end{align*}

where $\theta$ is an index of income inequality ($\theta \in (\frac{|W|}{|N|}, 1)$), $y$ is the total income in the economy.

There is a single political decision taken by the government, an income tax rate $\tau$, with the proceeds redistributed equally to all members of society. Taxation produces a deadweight loss of $C(\tau) = \frac{\tau}{|N|}$. We assume that this cost of taxation is null when there is no tax ($C(0) = 0$), increasing in the tax rate ($C'(\tau) > 0$) and increasing at an increasing rate ($C''(\tau) > 0$), with tax rates close to 1 being prohibitively inefficient ($C'(1) > 1$). Call $ny^\ell(i)(\tau)$ the net income of an individual $i$ when a tax rate $\tau$ is implemented. We have

\[ ny^\ell(i)(\tau) = (1 - \tau)y^\ell(i) + (\tau - C(\tau))\frac{y}{|N|} \]

There are two candidates $\ell$ running for election, one candidate represents the poor and is called the left candidate ($\ell = L; g(L) = P$) and another represents the wealthy and is called the right candidate ($\ell = R; g(R) = W$). The left candidate wants to maximize the utility of the poor, while the right candidate wants to maximize the utility of the wealthy. The utility of a candidate $\ell$ is, succinctly, $u^\ell(\tau) = ny^\ell(i)(\tau)$.

Citizens evaluate candidates on the tax policy that they expect them to implement and on a series of other dimensions. Let there be a aggregate valence shock $s$ for the right candidate (experienced by all citizens). Let $F(s)$ be the cdf of $s$. We assume that it is twice differentiable, with $f(s)$ the corresponding pdf, and impose the following condition:

**Condition 1** The hazard function $\frac{f(s)}{1 - F(s)}$ is increasing in $s$.

This is a relatively weak condition which holds for the logistic distribution, for example.

We assume that there is a single lobby, $Wl$, which represents the interests of the wealthy. We believe this assumption is reasonable for two reasons. First, the poor form a larger group and face a more severe collective action problem. Second, we are interested in the

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6 This set-up is taken from Acemoglu and Robinson (2006). We think that it is reasonable to assume that the marginal cost of taxation is increasing with the tax rate, either because higher tax rates have greater disincentives to effort or encourage greater levels of tax evasion. It would also be interesting to analyze richer tax schemes, if we allowed for greater variation in income. We leave this question for future research.

7 We could also assume that the candidate wants to maximize redistribution to his group and we would get the same results.
rise of the left, where incumbent parties are located on the right. If there is an advantage in the disbursement of clientelistic goods for the party in power, then this framework is useful in understanding the rise of left-wing governments. The lobby attempts to affect the election outcome through offering targeted benefits to voters or, more precisely, a bribing schedule \(b(.) : \{1, \ldots, |N|\} \rightarrow \mathbb{R}_+\). In this model, bribes are credible promises which are paid if and only if the right candidate is elected. We can interpret this set-up in different ways, each capturing some aspects of clientelism as surveyed in the literature. First, it captures the disbursement of patronage benefits, i.e., the transfer of government resources if and only if the right party is elected. Second, it captures a set-up where private goods are offered prior to the election and the lobby can costlessly punish every voter who received such bribes if the right party is not elected. Vote-buying is assumed to be an inefficient transaction, so that if the right candidate is elected, the total cost to the lobby is \(\gamma \sum_{i \in N} b(i)\), where \(\gamma \in \left(1, \frac{|P|}{|N| + |W|} \right)\).

The wealthy lobby maximizes the ex ante utility of all wealthy citizens minus the cost of providing bribes. Assuming that tax rates \(\tau^L, \tau^R\) are the tax rate proposed by the left and the right candidate, respectively, and the bribing schedule is \(b(.)\), then the lobby's payoff is:

\[
u^{Wl}(\tau^L, \tau^R, b(.)) = \sum_{i \in W} \left[ \int_{s : E = L} ny^W(\tau^L) dF(s) + \int_{s : E \neq L} \left( ny^W(\tau^R) + s + b(i) \right) dF(s) \right] - \sum_{i \in N, s : E \neq L} \gamma b(i) dF(s)
\]

where \(E \in \{L, R\}\) is the outcome of the election. Let \(v(i, b(i), s)\) be the voting decision of individual \(i\) as a function of the bribe offered to him by the lobby, \(b(i)\), and the aggregate preference shock \(s\). \(v(., ., .) : \{1 \ldots |N|\} \times \mathbb{R}_+ \times \mathbb{R} \rightarrow \{L, R\}\).

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8 We discuss these assumptions in the Latin American context in the Empirical Analysis section.

9 We should note that the main conclusion of the model, as outlined in Corollary 1, are robust to different specifications of vote-buying. A natural alternative would be to assume that bribes to individual \(i\) are contingent on individual \(i\)'s vote. This set-up raises some difficulties. First, it assumes that the lobby can monitor individual votes, which is strictly more difficult to achieve. Second, it generates multiple voting equilibria, as voters face a coordination problem when they are offered positive bribes but prefer the left candidate to be elected. Nevertheless, under some relatively general restrictions on the way that this coordination problem is solved, we obtain a unique equilibrium outcome, producing the result in Corollary 1. Also, we could model bribes as an investment in the electoral success of the right party, shifting the distribution of the preference shock \(s\) for the right candidate by \(b(i)\). In this case, the cost of a bribe would be paid up front by the lobby, but the benefits would accrue to the targeted voter only if the right candidate is elected. Corollary 1 continues to hold in that set-up. For an insightful discussion of these issues, see Dal Bo (2007).

10 We assume that \(\gamma > 1\) to ensure that there is a unique optimal bribing schedule and \(\gamma < \frac{|P|}{|N| + |W|}\) as a sufficient condition to our main corollary. If \(\gamma = 1\), we lose the uniqueness of the optimal bribing schedule, but we still obtain that there is a unique outcome of the voting game and the comparative statics with respect to inequality go through.
Timing

1. Wealthy lobby picks a bribing schedule $b(\cdot)$ to offer to voters
2. The valence shock $s$ is realized
3. Citizens pick their voting decision $v(\cdot, \cdot, .)$
4. Election outcome $E$ is realized
5. Elected candidate implements the tax policy $\tau$ and payoffs are realized

Solution Concept

We solve for a subgame-perfect equilibrium and we let voters pick weakly undominated strategies. Let $\{\sigma^W, \{\sigma^i\}^{N}_{i=1}, \sigma^L, \sigma^R\}$ be a strategy combination, where $\sigma^W = b(\cdot)$ is the bribing schedule by the wealthy lobby; $\{\sigma^i\}^{N}_{i=1}$ are the voting decisions for each individual, $\sigma^L = \tau^L$ and $\sigma^R = \tau^R$ are the tax policies of each candidate. The strategy combination $\{\sigma^W, \{\sigma^i\}^{N}_{i=1}, \sigma^L, \sigma^R\}$ is a subgame-perfect equilibrium if strategies form a Nash equilibrium in each proper subgame. Moreover, voting decisions are weakly undominated if they satisfy the following restriction. Write the net benefit of the left candidate for individual $i$, when he is offered a bribe $b(i)$ and the aggregate preference shock is $s$, as $n(i, b(i), s) = ny^{gL}(\tau^L) - ny^{gR}(\tau^R) - s - b(i)$. Then weakly undominated strategies require that $v(i, b(i), s) = L$ if and only if $n(i, b(i), s) \geq 0$.\(^{11}\) Write $^*$ for equilibrium strategies.

Solution of the Model

We solve the game backwards. The tax rate implemented by a candidate $c$ solves the following problem:

$$\tau^c = \arg \max_{\tau} ny^{gL}(\tau)$$

We get $\tau^R = 0$ since $\theta > \frac{|W|}{|N|}$, while $\tau^L$ satisfies

$$C'(\tau^L) = 1 - \frac{|N|}{|P|}(1 - \theta) \quad (1)$$

with the property that $\frac{\partial \tau^L}{\partial \theta} > 0$ since $C''(\cdot) > 0$.

Moving up, consider the voting decision. It is clear that there is a (generically) unique election outcome.

Claim 1 The left candidate is elected if and only if there is a majority of voters for whom $n(i, b(i), s) \geq 0$.

Proof: Obvious, since individual $i$ votes for candidate $c = L$ if and only if $n(i, b(i), s) \geq 0$. $\blacksquare$

\(^{11}\) Voting strategies are undetermined in the case where $n(i, b(i), s) = 0$, but there is no problem in assuming that the voter votes for the left candidate in this case, which happens with probability 0.
Moving up, consider the decision by the lobby to offer bribes. First, we can show the following claim:

**Claim 2** In any equilibrium, there is a set of poor voters $P_+$, such that the following conditions hold: (i) $\sum_{i \in N} b^\ast(i) = \sum_{i \in P_+} b^\ast(i)$, (ii) $|P_+| = \frac{|N|+1}{2} - |W|$, (iii) there is a $0 \leq b_P \leq \left[ny_P^L(\tau^L) - ny_P(0)\right] - \left[ny_W^L(\tau^L) - ny_W(0)\right]$ such that for all $i \in P_+$, $b^\ast(i) = b_P$.

**Proof:** See the Appendix.

This claim shows that, in any equilibrium, the lobby only buys off poor voters (i), the set of poor voters who receive bribes form a minimum-winning coalition with the wealthy (ii), and every voter who receives targeted benefits receive the same level of benefits (iii). The intuition behind these results is as follows. Given any bribing schedule, we can identify a set of median voters, who are such that the left candidate is elected if and only if they prefer the left candidate. Since bribes are not contingent on the preference shock $s$ and since this shock is an aggregate shock, the set of median voters is the same for any preference shock $s$. Then it is clear, first, that the lobby should pay bribes only to this set of median voters. Second, the cheapest way to construct such a set of median voters is to form a coalition with all the wealthy citizens and a sufficient number of the poor. Third, since every poor voter has the same income, any poor voter targeted by the lobby should receive the same level of benefits.

Now the question is: why does the wealthy lobby not offer any positive bribe to the wealthy voters? When the lobby offers small bribes to some poor voters, they constitute the set of median voters, since the wealthy care the most about getting the right candidate elected. Now, the lobby can make these poor voters care as much as wealthy voters about the election of the right candidate if they offer them a sufficiently high level of bribes. To further increase the probability that the right candidate is elected, the lobby should increase the bribes offered to these poor voters, but it should also offer some bribes to wealthy voters. Why are these bribes unacceptable? Because the lobby maximizes the ex ante utility of the wealthy voters. Indeed, offering bribes to wealthy voters affects the election outcome only when the preference shock is so much in favor of the left candidate that the wealthy voters would not want the right candidate to be elected (ex ante). Therefore, the lobby never offers positive bribes to wealthy voters.

This result is interesting in its own right given a consistent finding in the large literature on clientelism, which states that private benefits typically target the poor. Scholars have advanced various explanations for this phenomenon, including risk-aversion (Kitschelt, 2000; Wantchekon, 2008; see Nazareno et al. 2008 for doubts about such an approach) and diminishing marginal utility of income (Calvo and Murillo, 2004; Dixit and Londregan, 1996; see Stokes, 2008 for a review). The current set-up offers a simple explanation which shows that neither of these factors is necessary for the finding. If relatively affluent voters are organized and benefits are handed out to maximize their expected utility, then bribes will be targeted to poor voters.\(^\text{12}\)

\(^\text{12}\) An alternative explanation would be to assume that clientelism is targeted to buy turnout (Nichter, 2008). While this answer is certainly sensible, we believe that it has limited power in explaining...
Given the relatively simple structure of the optimal bribing schedule, we can show the following proposition:

**Proposition 1** There is a unique optimal bribing schedule \( b^*_P \) for any value of inequality \( \theta \). Moreover, there is a value \( \overline{\theta} \) such that for any \( \gamma > \overline{\theta} \), there is a value \( \overline{\theta} \in (\frac{|W|}{|N|}, 1) \) such that

(a) for any \( \theta \in \left[ \frac{|W|}{|N|}, \overline{\theta} \right], b^*(i) = 0 \) for all \( i \) and (b) for any \( \theta \in (\overline{\theta}, 1] \), there is a set \( P_+ \subset P \), with \( |P_+| = \frac{|N|+1}{2} - |W| \), such that

\[
b^*(i) = \begin{cases} b^*_P, & \text{for } i \in P_+ \\ 0, & \text{otherwise} \end{cases}
\]

where \( b^*_P \) is given implicitly by

\[
\left[ \frac{|N| + 1}{2} - |W| \right] \gamma \left[ (1 - F(s)) + f(s)b_P \right] = |W| \left[ ny^W(0) - ny^W(\tau^L) + \bar{s} \right] f(s) \tag{2}
\]

where

\[
\bar{s} = ny^P(\tau^L) - ny^P(0) - b_P
\]

**Proof:** See the appendix. \( \blacksquare \)

This proposition states that there is a unique optimal bribing schedule. The lobby always targets bribes to a minimum winning coalition, as shown in Claim 2. Whenever these bribes are positive, the optimal amount equates the marginal cost of a bribe (the left-hand side of (2)) to the marginal benefit of a bribe (the right-hand side of (2)). The marginal cost of a bribe takes into account the fact that bribes are targeted to a group of poor people of size \( \frac{|N|+1}{2} - |W| \), that they cost \( \gamma \) to provide due to inefficiency of vote-buying, and that they increase the total cost of vote-buying if marginally lower bribes would have been accepted (which happens with probability \( 1 - F(s) \)) or if they affect the election outcome (parametrized by the density \( f(.) \) evaluated at \( s \)). The marginal benefit of a bribe to the lobby sums over all wealthy citizens (\( |W| \)) the level of redistribution avoided (\( ny^W(0) - ny^W(\tau^L) \)) and the valence shock experienced (\( \bar{s} \)) if a marginal increase in bribes affects the election outcome (parametrized by the density \( f(.) \) evaluated at \( s \)).

It is clear that if inequality is low enough (\( \theta < \overline{\theta} \)), the wealthy lobby does not offer any bribe. To see this, note that the marginal benefit of a bribe converges to (at most) 0 when the distribution of income converges to perfect equality (\( \theta = \frac{|W|}{|N|} \)), since the right and left parties converge to the same proposed tax rate. The marginal cost of a bribe, however, is strictly bounded away from 0, since the wealthy lobby must target a strictly positive number of poor voters to affect the election outcome. Therefore, there are levels of inequality which are low enough that the wealthy lobby does not offer any bribe.

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the rise of the left, with the party on right initially in power and benefiting from the strongest clientelistic networks. Moreover, we do not want to rule out vote-buying from sources other than political parties, and we contend that the wealthy should have the advantage in vote-buying from these sources. In that case, we believe that our set-up would be more appropriate.
Now we ask whether the wealthy lobby does offer some bribes in equilibrium. This will depend on whether there are sufficiently high stakes in the election. As the level of income increases, avoiding the tax rate preferred by the poor becomes more attractive to the wealthy (as it prevents a greater amount of redistribution). Therefore, there is a value of income sufficiently high \((y > \bar{y})\) that the wealthy lobby does offer bribes in equilibrium, if inequality is sufficiently high \((\bar{\theta} < 1)\).

Now turning our attention to the comparative statics, we note the following result:

**Corollary 1** The probability that the left candidate is elected is increasing with inequality when inequality is below the cut-off \(\bar{\theta}\) and decreasing with inequality when inequality is above the cut-off \(\bar{\theta}\). In other words, \(\frac{\partial \text{prob}(E=L)}{\partial \theta} \geq 0 \iff \theta < \bar{\theta}\)

**Proof:** See the Appendix.

This corollary gives us the main prediction of the model: an inverted U-shaped relationship between inequality and the probability that the left candidate is elected. See Figures 1 and 2 for a graphical illustration. Figures 1 and 2 plot, on the horizontal axis, a voter index, going from 1 to \(N\). On the vertical axis, we plot the net benefit of voter \(i\) for the left candidate, at a valence shock \(s = 0\). A positive preference shock \(s\) shifts the net benefit of any voter \(i\) for the left candidate upward by \(s\). Absent any bribe, wealthy voters have a lower net benefit for the left candidate than poor voters. Offering bribes to voter \(i\) shifts downwards the net benefit of \(i\) for the left candidate. A voter prefers to vote for the left candidate, given some bribes and a preference shocks \(s\), if and only if he has a positive net benefit for the left candidate (and for that reason, we plot the point 0 on the vertical axis). As a result, a voter targeted by a bribe prefers the left candidate for a smaller set of preference shocks \(s\).

In Figure 1, we plot the situation where inequality is low and, consequently, the lobby does not offer any bribe to poor voters. As inequality increases, the amount of redistribution generated by the left candidate increases and, as a result, wealthy voters
Figure 2. Net benefit of voters for the left candidate: Capturing the effect of increased inequality (from a high level of inequality).

have a lower net benefit for the left candidate, while poor voters have a greater net benefit for the left candidate. Since the poor voters form a majority, the probability that the left candidate is elected increases with inequality.

In Figure 2, we plot the situation where inequality is high (above $\bar{\theta}$). In such a case, the lobby pays positive bribes to some poor voters. The net benefit of the left candidate for any other voter, who does not receive positive bribes, follows the same dynamic as in Figure 1. How does the net benefit of the left candidate for the poor voters, targeted by the lobby, vary with inequality? We know that the poor, absent any bribe, have a greater preference to get the left candidate elected. But we also know that the wealthy have a greater preference to get the right candidate elected, which should increase the level of bribes chosen by the lobby. Which effect dominates? Given that taxes are inefficient, the cost of redistribution for the wealthy is greater than the benefit of redistribution for the poor. Therefore, the wealthy voters’ resistance to inequality dominates.\(^{13}\) Graphically, the net benefit of the left candidate for the poor voters targeted with bribes goes down with inequality. As a result, as inequality increases, the probability that the left candidate is elected decreases.

EMPIRICAL ANALYSIS

We now take the model developed in the previous section to the data. While the model is general, we believe that it is particularly appropriate for explaining the rise of the left

\(^{13}\) This result assumes that vote-buying is not too inefficient, i.e., $\gamma < \frac{|P|}{N+1/2} - |W|$, which ensures that the cost of bribing a minimum number of voters is less than the value of redistribution for all the poor voters. We discuss this assumption further in the conclusion.
In Latin America, given the region’s unique combination of democracy, relatively weak rule of law (so that vote buying is possible) and moderate levels of income (so that there is enough sufficiently at stake such that vote buying may happen in equilibrium, as seen in Proposition 1).

Our model explores the specific contention that the right has an advantage in vote buying, particularly when it is already in power. How well does this assumption fit reality? To be sure, recent research on Latin America points to relatively large clientelist networks for parties on the left, at least when they are in power (e.g. Calvo and Murillo, 2009). But, other evidences support a claim closer to ours that leftist parties have frequently been at a disadvantage in vote buying, especially when they were out of power. Lyne (2008) describes the rise of Hugo Chávez in Venezuela and Luis Ignacio Lula da Silva in Brazil in exactly those terms. In turn, Luna (2007) documents how, in Uruguay, the (leftist) Frente Amplio broke 175 years of electoral dominance by the Colorado and Blanco parties, which had established extensive clientistic networks. Moreover, in cases where the right successfully continued its electoral dominance, its success is partly attributed to its effectiveness at providing clientistic goods (Middlebrook, 2000, p. 41). And even where the right has been out of power it may still have the advantage in vote buying, given its closer connection to wealthy interests. For example, Luna (2006, pp. 447–448) documents how the (right-wing) UDI in Chile “benefits extensively from being the major opposition party, while commanding at the same time, the control of private resources to be invested in clientelistic politics.” However, to allow for the possibility of the party on the right has the advantage in vote buying only when it is in power, we will conduct our tests on two samples, first using all elections and then by focusing on cases where the right is in power.

14 Models with multiple lobbies are typically cumbersome (see Groseclose and Snyder, 1996 for a model of sequential vote-buying).

15 Note, however, that the two most studied clientelistic parties in Latin America, the Argentine Peronist Party and the Mexican PRI party, are notoriously hard to place ideologically (see Coppedge, 1997). Indeed, observers point out that clientelism in Argentina became more prevalent as the Peronist party moved to the right, pursuing neoliberal policies under Menem (cf. Stokes et al., 2004, p. 73).

16 The seven country cases in his edited volume are Chile, Colombia, Venezuela, Argentina, Brazil, El Salvador and Peru. For example, Middlebrook (2000, p. 40) attributes the success of the (right-wing) ARENA party in El Salvador, in the 1980s and 1990s, in part, to its ability to gain significant electoral support in rural areas through traditional patron–client ties. Dugas (2000) argues that, in Colombia, the (right-wing) Partido Conservador and Partido Liberal both developed extensive networks of broker clientelism in the National Front regime, between 1958 and 1974.

17 We purposely use a general term for the identity of the wealthy vote buyer, so as to allow, for example, private firms to pressure their employees to support the candidate of the right. This assumption was appropriate, for example, in Brazil, where employers’ associations served as useful intermediates to the government in buying votes from their employees (Lyne, 2008, p. 80). The model can also accommodate a situation where political parties are the direct entity purchasing votes.
Data

To examine whether our theoretical predictions are borne out empirically, we have assembled a dataset of 110 elections covering 18 countries in Latin America, spanning the entire third wave democratization period from 1978 to 2008. Our dependent variable, $\text{Left}_i,t$, is a dummy variable designed to capture the ideology of the elected president in a given country $i$ and year $t$.\(^\text{18}\)

We take two measures of the ideological orientation of presidents. First, we identify the pure left. This is the set of presidents, many of whom have attracted worldwide attention lately, with platforms clearly centered around redistribution. See Table 1 for a list of these presidents.

Second, we use the relative left presidents, who were considered the most redistributive among the top three vote-getters in the election, as coded by Stokes (2009).\(^\text{19}\)

Consistent with the narrative on the rise of the left that dominates the literature, most of the pure left presidents were elected (and some re-elected) since 1998. The four exceptions to this pattern are Alan García, who was elected President of Peru in 1985, Daniel Ortega, leader of the FSLN revolutionary party, elected as President in Nicaragua in 1984, Jaime Paz Zamora, elected President of Bolivia in 1989, and Rodrigo Borja, elected President of Ecuador in 1988.\(^\text{20}\)

Perhaps the biggest empirical challenge to testing our theory has been to identify adequate and comparable measures of inequality, given the diversity in the definitions of income and units of analysis in different surveys. We construct our measures of inequality from the United Nations University’s World Institute for Development Economics Research (UNU-WIDER). This database builds on the former Deininger and Squire study and collects measures of inequality from a wide range of sources. From these data, we constructed our main independent variables, $\text{Inequality}_i,t-1$ and $\text{Inequality}_i,t-1^2$, by taking the inequality measure from the closest year before any election year $t$ in

\(^{18}\) To code the left, we used a variety of sources on party ideology in Latin America including Coppelde (1997), when available, Castaneda (2006), Cleary (2006), Murillo et al. (2008), Stokes (2009) and Weyland (2008). In a few cases where our coding differs from Murillo et al.’s (i.e., they code Presidents N. Kirchner, C. Kirchner, Borja, Paz Zamora, and Garcia as center-left as opposed to left), we have re-run the analysis switching the coding to center-left for each of the presidents and find no substantive difference with our main results presented below. Note that we also focus on the election of left candidates, as opposed to the vote share of the left candidate, as it is the main variable of interest in constructing the model and establishing the comparative statics.

\(^{19}\) We completed the coding for missing observations from her sample. The missing observations were Neves (Brazil, 1985), Balaguer (Dom. Rep., 1986, 1990, 1994), Lucas Garcia (Guatemala, 1978), Anibal Garcia (Guatemala, 1982), Ortega (Nicaragua, 1984), Endara (Panama, 1989), Lugo (Paraguay, 2008), and Fujimori (Peru, 1995). Only Ortega (Nicaragua, 1984) and Lugo (Paraguay, 2008) were considered relative left according to this definition. The list of relative left presidents is available upon request. Obviously, with this broader definition, there were more relatively left (60) than pure left presidents (19).

\(^{20}\) Note that we have not included Alan Garcia’s second government (2006–present) as a pure left government. In this second election, Ollanta Humala was the leftist candidate, while Garcia moved more the center.
Table 1. Pure left presidents in Latin America, 1978–2008.

<table>
<thead>
<tr>
<th>Country</th>
<th>Year</th>
<th>President</th>
<th>Party</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argentina</td>
<td>2003</td>
<td>N. Kirchner</td>
<td>PJ/FPV</td>
</tr>
<tr>
<td>Argentina</td>
<td>2007</td>
<td>C. Kirchner</td>
<td>PJ/FPV</td>
</tr>
<tr>
<td>Bolivia</td>
<td>1989</td>
<td>Paz Zamora</td>
<td>MIR</td>
</tr>
<tr>
<td>Bolivia</td>
<td>2005</td>
<td>Morales</td>
<td>MAS</td>
</tr>
<tr>
<td>Brazil</td>
<td>2002</td>
<td>Lula</td>
<td>PT</td>
</tr>
<tr>
<td>Brazil</td>
<td>2006</td>
<td>Lula</td>
<td>PT</td>
</tr>
<tr>
<td>Chile</td>
<td>2000</td>
<td>Lagos</td>
<td>Socialist</td>
</tr>
<tr>
<td>Chile</td>
<td>2006</td>
<td>Bachelet</td>
<td>Socialist</td>
</tr>
<tr>
<td>Dom Republic</td>
<td>2000</td>
<td>Mejia</td>
<td>DRP</td>
</tr>
<tr>
<td>Ecuador</td>
<td>1988</td>
<td>Borja</td>
<td>ID</td>
</tr>
<tr>
<td>Ecuador</td>
<td>2006</td>
<td>Correa</td>
<td>Alianza Pais</td>
</tr>
<tr>
<td>Nicaragua</td>
<td>1984</td>
<td>Ortega</td>
<td>FSLN</td>
</tr>
<tr>
<td>Nicaragua</td>
<td>2006</td>
<td>Ortega</td>
<td>FSLN</td>
</tr>
<tr>
<td>Paraguay</td>
<td>2008</td>
<td>Lugo</td>
<td>Patriotic Alliance for Change</td>
</tr>
<tr>
<td>Peru</td>
<td>1985</td>
<td>Garcia</td>
<td>APRA</td>
</tr>
<tr>
<td>Uruguay</td>
<td>2004</td>
<td>Vazquez</td>
<td>Frente Amplio</td>
</tr>
<tr>
<td>Venezuela</td>
<td>1998</td>
<td>Chavez</td>
<td>MVR</td>
</tr>
<tr>
<td>Venezuela</td>
<td>2000</td>
<td>Chavez</td>
<td>MVR</td>
</tr>
<tr>
<td>Venezuela</td>
<td>2006</td>
<td>Chavez</td>
<td>MVR/PSUV</td>
</tr>
</tbody>
</table>

More precisely, we use three different measures of inequality. With a first look at the data, we take any measure of Gini coefficients, the most standard measure of inequality. Next, we restrict our attention to Gini coefficient based on surveys of gross income. This is obviously a more appropriate measure, but it means that many measures of income are dropped. Finally, we use the share of (gross) income owned by the top

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21 If there are no measures available prior to the election, we take the closest future measure of inequality (this occurred for only one country, as Nicaragua held elections in 1984 and 1990 but the first available measure of inequality is from 1993). If there is more than one measure for inequality for a given country year, we take the average.

22 More specifically, we drop any survey of Monetary Income, Disposable, Earnings, Net, Income, Disposable, Consumption and Expenditure. An alternative approach would be to standardize the measures of inequality based on different concepts of income. The Standardized Income Inequality Data (SIDD), described in Babones and Alvarez-Rivadulla (2007), tackles such issues of comparability, but we have decided against using its latest version (SIDD-3) for the same reason that we did not use the data on labor shares. For 8 of the 18 countries, SIDD-3 reports a single value of inequality for the whole time period; for all the remaining countries, the Gini coefficient is extrapolated (and staying constant) from about the mid-1990s onwards. The cost in terms of loss of information is too large for our study, as we try to explain the rise of the left in the last decade.
quintile of the income distribution. This is a particularly close measure of \( \theta \), the share of income owned by the rich.\textsuperscript{23}

Latin America, as many observers have previously noted, is the most unequal region in the world (Karl 2003). Indeed, it is for this very reason that scholars often consider inequality as one of the main underlying structural conditions for the left’s success in Latin America (Castañeda, 2006; Cleary, 2006; Levitsky and Roberts, 2008). Yet, the key point here is that inequality also varies substantially within Latin America both over time and, most especially, across countries. This is clear from looking at Table 2, which reports summary statistics about the level of inequality at the time of elections, listing the mean levels of inequality and the standard deviation of inequality for each country, for each measure of inequality used in this study.

Thus, while the mean Gini for the region is 50.62, there are large differences between a country of relatively low inequality (Uruguay, for example, with a mean Gini of 42.20) and relatively high inequality (Brazil, for example, with a mean Gini of 58.96). A country

<table>
<thead>
<tr>
<th>Country</th>
<th>Gini using all measures of income</th>
<th>Gini using gross measures of income</th>
<th>Top quintile share of income using gross measures of income</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Average inequality</td>
<td>Standard deviation</td>
<td>Average inequality</td>
</tr>
<tr>
<td>Argentina</td>
<td>47.00</td>
<td>4.48</td>
<td>47.31</td>
</tr>
<tr>
<td>Bolivia</td>
<td>52.61</td>
<td>3.79</td>
<td>52.88</td>
</tr>
<tr>
<td>Brazil</td>
<td>58.96</td>
<td>1.66</td>
<td>58.96</td>
</tr>
<tr>
<td>Chile</td>
<td>54.48</td>
<td>1.95</td>
<td>54.21</td>
</tr>
<tr>
<td>Colombia</td>
<td>54.34</td>
<td>5.16</td>
<td>54.34</td>
</tr>
<tr>
<td>Costa Rica</td>
<td>46.87</td>
<td>1.62</td>
<td>46.77</td>
</tr>
<tr>
<td>Dominican Republic</td>
<td>48.43</td>
<td>3.30</td>
<td>47.82</td>
</tr>
<tr>
<td>Ecuador</td>
<td>53.88</td>
<td>8.23</td>
<td>58.89</td>
</tr>
<tr>
<td>El Salvador</td>
<td>48.67</td>
<td>4.80</td>
<td>48.33</td>
</tr>
<tr>
<td>Guatemala</td>
<td>49.04</td>
<td>8.52</td>
<td>50.6</td>
</tr>
<tr>
<td>Honduras</td>
<td>53.85</td>
<td>0.67</td>
<td>53.01</td>
</tr>
<tr>
<td>Mexico</td>
<td>52.38</td>
<td>1.96</td>
<td>49.1</td>
</tr>
<tr>
<td>Nicaragua</td>
<td>53.55</td>
<td>0.73</td>
<td>55.18</td>
</tr>
<tr>
<td>Panama</td>
<td>53.75</td>
<td>4.14</td>
<td>53.65</td>
</tr>
<tr>
<td>Paraguay</td>
<td>50.15</td>
<td>7.37</td>
<td>57.27</td>
</tr>
<tr>
<td>Peru</td>
<td>50.53</td>
<td>7.46</td>
<td>54.64</td>
</tr>
<tr>
<td>Uruguay</td>
<td>42.20</td>
<td>2.68</td>
<td>42.02</td>
</tr>
<tr>
<td>Venezuela</td>
<td>44.05</td>
<td>4.26</td>
<td>49.24</td>
</tr>
</tbody>
</table>

\textsuperscript{23} We thank the editors for suggesting that we try alternative measures of inequality.
with average inequality, for example Paraguay, may nevertheless see wide variation in inequality over time (around a mean level of 50.15, with a standard deviation of 7.37).24

Bearing out the overall claim that the relationship between inequality and democracy is more complex than commonly noted, countries with the lowest average overall levels of inequality include not only some of the countries considered among the most stable and prosperous in the region, such as Uruguay and Costa Rica, but also some of the region’s most troubled cases, including Venezuela, El Salvador, and Guatemala. Likewise, at the highest levels of inequality we find some of the region’s wealthiest and highest performing democracies (Chile and Brazil) alongside of some of the poorest and least functional governments (Ecuador and Honduras).

In the vast majority of these countries, just as critics of neo-liberalism claim, inequality rose sharply in the mid- and late 1990s.25 Yet we also see that inequality within the region has also converged in the last decade. In other words, inequality has decreased in high-inequality countries (such as Brazil) and increased in low-inequality countries (such as Argentina and Uruguay; see Figure 3).

Moreover, in most of these cases the relationship between the left winning and inequality is not monotonic. Rather in the majority of countries the left wins when inequality levels are already relatively low compared to peak levels. In Brazil and Chile, centrist incumbent governments had already started to bring inequality levels down from record highs prior to the left coming to power. Likewise, in Paraguay, Guatemala, Costa Rica, Bolivia, and Ecuador inequality was already trending downward by the time the left won power. While the descriptive patterns of the relationship between the leftist turn and inequality for individual countries are thus roughly consistent with the inverted U-shaped pattern we predict, we now turn to regression analysis to test systematically our theory. As part of this effort, we also include a host of additional variables to control for several alternative explanations for the left’s success.

Alternative Hypotheses and Control Variables

Economic Voting

One of the most compelling alternative accounts connects the recent success of the left in Latin America to a standard retrospective theory of voting behavior (e.g., Cleary, 2006;
Levitsky and Roberts, 2008; Murillo et al., 2008; Panizza, 2005). Following this logic, Levitsky and Roberts (2008) point out that the economic downturn in Latin America between 1998 and 2002 led voters to reject incumbents. In their words, “Latin America’s Left turn may be as much a product of anti-incumbent voting as it was a product of ‘Left’ voting” (Ibid. 2008, p. 25). Because most incumbents in the region were right or center-right, this ushered in an era of new left-wing governments, many of which have been re-elected subsequently due to the economic turnaround in the region post-2003 (Ibid. 2008, pp. 24–26).

In this vein, Murillo et al. (2008) ask whether the likelihood of the left vote share increases in response to the economic successes or failures achieved by a right incumbent government (incumbent performance). Their evidence is mixed. The inability of right incumbents to control inflation benefits the left, but so, it seems, does high growth produced by right governments (Ibid. 2008, p. 25). Thus, they are left with the intriguing result that voters in Latin America punish rightist incumbents for high inflation, but do not reward them for high growth.

We control for these factors, employing two economic variables, taken from the World Bank. Growth$_{it-1}$ is the lagged value of the growth of per capita GDP in constant 2000 US$. Inflation$_{it-1}$ is the lagged value of inflation (GDP Deflator).

**Party System and Mass Mobilization**

Another explanation for the rise of the left takes a more historical view, focusing on the legacy of labor and the capacity for mass mobilization. Building on a pioneering study
by Roberts (2002), which distinguishes between elite-mobilizing and mass-mobilizing party systems, Cleary (2006) argues that the recent success of the left in Latin America has largely been confined to countries with a tradition of mass-mobilizing party systems (Argentina, Bolivia, Brazil, Chile, Peru, and Venezuela). The logic of the argument is that even in contexts where the left has been weakened (either through direct military repression in the 1970s or through the indirect vagaries of the market economy in the 1980s and 1990s) the new post-2000 left is able to draw on some latent capacity among the poorer sectors for organization and mobilization.

To explore whether the party system legacy affects the likelihood that the left will win power, we construct the dummy variable Mass Mob., which is coded 1 for countries Roberts (2002) identifies as mass-mobilizing (Argentina, Bolivia, Brazil, Chile, Mexico, Nicaragua, Peru, Venezuela, El Salvador and Guatemala) and 0 for countries identified as elite-mobilizing (Colombia, Costa Rica, Dominican Republic, Ecuador, Honduras, Panama, Paraguay, Uruguay).26

**Age of Democracy and Cold War**

Finally, scholars have posited two other factors that would explain the recent success of the left: the age of a democracy and the end of the Cold War. First, the left may be more successful in mature and consolidated democracies. Chávez aside, one of the most prominent features of the left in Latin America today is moderation. A recent paper by Weyland (2008) underscores the relative continuity between the neo-liberal economic policies implemented by the right and those being implemented by today’s left. Consider the case of Chile. Lagos and Bachelet may share the same party as Allende, but both Presidents have been wholly committed to the market model implemented under Pinochet and attempt to soften the “hard edges” of capitalism mostly by using a mix of different social policies (*Ibid.* 2008, p. 17). Likewise, despite the fierce anti-neoliberal rhetoric of his campaign, current Ecuadorian President Rafael Correa has thus far refused to end dollarization. Weyland points out that even Chávez, with his supposed commitment to twenty-first century socialism, has not enshrined redistributionary policies into law, using them instead as a source of patronage to generate votes (*Ibid.* 2008, p. 9).

While the moderation of the new left may disappoint some stalwart leftist ideologues, others interpret it as sign of healthy democracy. Drawing on Boix’s (2003) logic about the constraints that mobile capital impose, Cleary (2006) notes that today’s left in Latin America is far less threatening to the right and, as a result, far more appealing to citizens who earlier rightly feared that electing the left was tantamount to triggering a military coup.

In the same vein, Castañeda (2006) notes the transformation of the left following the end of the Cold War. The left in Latin America today is surely less associated with the Russian Revolution, than with Western European social democracy. As Murillo *et al.*

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26 Note that Roberts (2002) does not include El Salvador and Guatemala, but to avoid losing observations from our dataset, we have coded them as elite-mobilizing countries. Our results, reported below, however, are robust to specifications that exclude both the countries.
Debs and Helmke (2008) suggest however, the process of incorporating the left into peaceful democratic politics is also linked with the amount of time that each country has spent under democracy. Perhaps because citizens and elites in older democracies have had more time to update their beliefs about the relatively moderate nature of the post-Third Wave left, the expectation is that the left is more likely to win votes as democracy ages.

To control for these influences, we construct the continuous variable “Age of Democracy, $i,t$,” which records the number of years that each country has spent under democracy since the last democratic transition for a given election year, following Boix (2003) and Przeworski et al. (2000).

We assess the impact of the international context on the probability of the left winning by creating a dummy variable, Cold War, which is coded 1 for elections prior to 1990, and 0 for elections from 1990 onwards.

Analysis

Tables 3 and 4 present a series of models used to test our hypotheses alongside several alternative explanations. We first discuss the evidence for the elections in which pure left candidates were elected, Models 1–4.

A central implication from our theory concerns the effect that the level of inequality has on the probability of the left winning any given country $i$ at year $t$. Specifically, we predict an inverted U-shaped relationship where the probability of a pure leftist president getting elected increases in inequality up to a point where the rich face incentives to bribe poor voters and decreases thereafter. With a dichotomous dependent variable, we run a maximum likelihood estimation of the following logit model, $\text{prob(} \text{left elected}) = \frac{\exp(\text{X}\beta)}{1+\exp(\text{X}\beta)}$, where

$$X\beta = \beta_0 + \beta_1\text{Inequality}_{i,t-1} + \beta_2\text{Inequality}^2_{i,t-1} + \beta_3 - \text{Control Variables}_{i,t}$$

The evidence would be consistent with our hypothesis if the sign of the coefficient for $\text{Inequality}_{i,t-1}$ is positive and the sign for $\text{Inequality}^2_{i,t-1}$ is negative, indicating an inverted U-shaped effect.

In each and every model contained in Table 3 our expectations are borne out. Model 1 serves as the baseline. Here, we employ a standard logit model for all elections contained in the data set using just our two key independent variables, $\text{Inequality}_{i,t-1}$ and $\text{Inequality}^2_{i,t-1}$. Substantively, the model shows that the probability of electing a pure

Note that these two sources differ for only three cases. Peru is considered democratic in the 1990s by Boix (2003), but not by Przeworski et al. (2000), due to their implementation of the alternation rule. Since we focus on the electorate’s expectation at the time of the election, we do include these elections. For the same reason, we place the start of democracy in Panama to 1989, when Guillermo Endara was elected president, though the election was briefly contested by Manuel Noriega (Przeworski et al. 2000 place the start of democracy in 1990 and Boix, 2003 in 1991). Finally, Paraguay is considered authoritarian by Przeworski et al. (2000) past 1996, while Boix (2003) declares it dictatorial until 1994. We depart from these sources and, consistent with Murillo et al. (2008) and Stokes (2009), we place the transition to democracy to 1989, when the first elections were held after the overthrow of long-time dictator Alfredo Stroessner Matiauda.
Table 3. Probability of success of pure left in presidential elections.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Inequality(_{i,t-1})</td>
<td>1.63 <em>(0.76)</em>*</td>
<td>2.16 <em>(1.12)</em></td>
<td>3.34 <em>(1.46)</em>*</td>
<td>4.59 <em>(2.50)</em></td>
</tr>
<tr>
<td>Inequality(_{i,t-1}^2)</td>
<td>-0.02 <em>(0.01)</em>*</td>
<td>-0.02 <em>(0.01)</em></td>
<td>-0.03 <em>(0.01)</em>*</td>
<td>-0.04 <em>(0.02)</em>*</td>
</tr>
<tr>
<td>Cold War(_i)</td>
<td>0.34 *(0.84)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age of Democracy(_i,t)</td>
<td>0.05 *(0.01)</td>
<td>0.35 *(0.01)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inflation(_{i,t-1})</td>
<td></td>
<td></td>
<td></td>
<td>-0.01 *(0.01)</td>
</tr>
<tr>
<td>Growth(_{i,t-1})</td>
<td></td>
<td></td>
<td></td>
<td>-0.32 *(0.20)</td>
</tr>
<tr>
<td>Mass Mob(_i)</td>
<td>2.29 <em>(0.88)</em>*</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>-43.80 <em>(18.72)</em>*</td>
<td>-59.92 <em>(30.74)</em>*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>110</td>
<td>110</td>
<td>69</td>
<td>58 (no left incumbent)</td>
</tr>
</tbody>
</table>

\(^* p < 0.10; ** p < 0.05; *** p < 0.01\) (Two-tailed test).

Standard errors in brackets, with robust standard errors, clustered by country, for the logit (Models 1 and 2).

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leftist president rises in inequality up to a Gini of 52 and then declines as inequality continues to rise. Specifically, at the 10th percentile of the distribution of inequality (a Gini of 42), the probability of electing a pure leftist president is 3%, rising to a maximum value of 13% at a Gini of 52, and declining to 9% at the 90th percentile of the distribution of inequality (a Gini of 58). Calculating the marginal effects, we find that at the 10th percentile, a one unit increase in the Gini coefficient leads to a 1.7% increase in the probability of a pure leftist president being elected, with a standard error of 0.007, while at the 90th percentile, a one unit increase in the Gini coefficient leads to a 2.4% reduction in the probability of a pure leftist president being elected, with a standard error of 0.019.

We then test our hypotheses against other explanations. Model 2 adds the controls Cold War\(_i\), Age of Democracy\(_i,t\), and Mass Mob\(_i\). We find that Inequality\(_{i,t-1}\) continues
Table 4. Probability of success of relative left in presidential elections.

<table>
<thead>
<tr>
<th>Model</th>
<th>Independent Variables</th>
<th>Coefficient</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 5</td>
<td>Gini-All</td>
<td>0.73</td>
<td>(0.36)**</td>
</tr>
<tr>
<td>Model 6</td>
<td>Gini-Gross</td>
<td>1.55</td>
<td>(0.87)*</td>
</tr>
<tr>
<td>Model 7</td>
<td>Gini-Gross</td>
<td>1.45</td>
<td>(0.82)*</td>
</tr>
<tr>
<td>Model 8</td>
<td>Top Quintile-Gross</td>
<td>4.12</td>
<td>(1.62)**</td>
</tr>
<tr>
<td>Model 9</td>
<td>Top Quintile-Gross</td>
<td>2.81</td>
<td>(1.93)</td>
</tr>
</tbody>
</table>

\[ \text{Inequality}_{t-1} = -0.01 (0.00)** \]
\[ \text{Inequality}_{t-1}^2 = -0.02 (0.01)* \]
\[ \text{Cold War} = 0.09 (1.37) \]
\[ \text{Age of Democracy}_{t} = (0.07) \]
\[ \text{Inflation}_{t-1} = 0.00 (0.00) \]
\[ \text{Growth}_{t-1} = -0.06 (0.10) \]
\[ \text{Constant} = -16.75 (8.46)** \]

\[ N = 110 \]
\[ \text{(no left incumbent)} = 104 \]
\[ \text{(no left incumbent)} = 57 \]

\[ ^{*} p < 0.10; ^{**} p < 0.05; ^{***} p < 0.01 \text{ (Two-tailed test)} \]

Standard errors in brackets, with robust standard errors, clustered by country, for the logit (Model 5).

To have the effect predicted by our theory.\(^28\) Moreover, the effect of Cold War\(_t\) and Age of Democracy\(_{i,t}\) on the election of pure left candidates is not statistically significant, while countries with a history of mass mobilization do seem to be more conducive to the election of left candidates. However, we find it difficult to assign to Mass Mob\(_i\) a causal effect on the election of left candidates. From a theoretical standpoint, it remains to explain both why and how either class cleavages suddenly emerged in traditionally elite mobilizing systems (such as Ecuador) or why the left manages to attract votes in the apparent absence of class cleavages. From an empirical standpoint, it is difficult to assess such a characteristic independently of the hindsight of electoral performances.

\(^{28}\) Adding these controls reduces the magnitude of the marginal effects, but they exhibit the same pattern (they are equal to 0.4% and −0.9%, with standard errors of 0.002 and 0.005, respectively, holding Age of Democracy\(_{i,t}\) at its mean and the Cold War\(_t\) and Mass Mob\(_i\) dummies at their median, i.e., 0).
Instead, we prefer to run a conditional fixed effect logit model to control for country-specific unobserved sources of heterogeneity.\textsuperscript{29} Our theory continues to be supported in the data (Model 3). By the same token, we find no evidence for the economic voting hypothesis.\textsuperscript{30} Model 4 restricts our attention to the set of elections with the left out of power. Consistent with the finding in Murillo \textit{et al.} (2008), it does not appear that the left reaps any noticeable benefits when non-left incumbents fail to deliver growth. Contrary to Murillo \textit{et al.} (2008), it does not appear that high inflation under a non-left incumbent helps the electoral prospects of the left either: the sign for $\text{Inflation}_{i,t-1}$ is in the wrong direction and does not achieve statistical significance.\textsuperscript{31}

The effect of the temporal controls is more puzzling. As expected, older democracies are more likely to elect pure left presidents. However, we find some evidence which seems to sharply contradict the Cold War hypothesis. The sign of the coefficient of Cold War$_i$ is positive and significant, suggesting that pure left governments were more likely to be elected during the Cold War. Yet we hesitate to put too much stock in this result. Specifically, the fixed effects model estimates within-country variation, dropping countries that elected no left government during the entire sample. For example, two of the oldest democracies, Costa Rica and Colombia, did not elect a pure left government throughout the period. Removing these countries from the sample exaggerate the odds of the election of the left during the Cold War and overstates the effect of the variable Age of Democracy$_i$.

Moreover, the effect of Cold War, may be confounded by the effect of Age of Democracy$_i$. After all, pure left presidents were elected only in the second half of the post-Cold War period.

Turning to Table 4, we check further the robustness of our empirical results by broadening our definition of leftist governments, and by refining our measure of inequality.

Starting with model 5, we replicate our baseline model using Stokes’s (2009) measure, relative left, as our new dichotomous dependent variable. Despite the fact that proportionately relative left governments are far more common than pure left governments, the evidence continues to support our main theoretical argument. That is, Inequality$_{i,t-1}$ exerts a nonlinear inverted U-shaped effect on the probability that a relative left government will come to power.

In Models 6–9, we examine whether our results continue to hold when we use different measures for inequality. Specifically, models 6 and 7 re-estimate the conditional fixed effects models for the relative left without and with controls using a measure of inequality that only considers surveys reporting gross pre-tax income. Finally, Models 8 and 9 replicate this same test using just the top quintile gross inequality measures. In each and every one of these models, the signs of the coefficient for Inequality$_{i,t-1}$ are in the

\textsuperscript{29} Countries which have never elected a left president are dropped, which explains the reduction of the sample from 110 to 69.

\textsuperscript{30} Unfortunately, a drawback of the conditional (fixed-effects) logit model is that we cannot compute the partial effects of independent variables. The model assumes the presence of country-specific parameters $c_i$, but does not estimate these parameters, as they drop out of the conditional maximum likelihood function (Wooldridge, 2002, pp. 490–492).

\textsuperscript{31} A reason that our results differ from the previous literature on the effects of inflation and growth may also lie in our dichotomous coding of the dependent variable (cf. Murillo \textit{et al.}, 2008).
expected direction, achieving statistical significance in three out of the four models.\textsuperscript{32} By contrast, none of the alternative explanations receive strong support, though now at least the signs for Inflation_{i,t−1}, Growth_{i,t−1}, Age of Democracy_{i,t}, and Cold War_{t} are in the expected direction.

In sum, regardless of how we measure the left or inequality, whether we include fixed effects or not, whether we include all elections or limit our sample to non-left incumbents, or whether we control for alternative economic or historical factors, we find striking and consistent empirical evidence that inequality shapes the fortunes of the left in precisely the way our theory predicts.

CONCLUSION

Inequality under democracy shapes the electoral fortunes of the left. Most scholars would surely agree with this statement, at least in the broadest sense. But, to date, neither the theory nor the facts in support of this claim have been particularly clear. Perhaps as a result, the new literature on the left in Latin America tends to treat inequality as an underlying constant, and not as a particularly useful variable for explaining when, where, or why the left wins or loses.

This paper takes a different tack. On a theoretical level, we supply a clear set of micro-foundations linking income distribution to the left’s chances for success. Using a comparative statics approach, we show that inequality has a non-monotonic effect on the left’s likelihood of success. At low levels of inequality the poor have relatively too little to gain in electing the left. At high levels of inequality the rich have too much to lose. Thus, we argue, the left is most likely to win office if inequality is somewhere in-between the two extremes. On an empirical level, we then bring fresh systematic evidence to bear both on our own prediction and several alternative hypotheses. Overall, our findings underscore that while other factors — such as the legacy of mass mobilization, the age of democracy and the end of the Cold War — appear to matter, only the level of income distribution has a consistent effect on whether the left succeeds or fails.

In identifying the mechanism through which inequality influences electoral outcomes under democracy, our theory also generates several additional questions for future research. First, it would be interesting to see whether the results hold in other weak democracies around the world. Second, it would be interesting to explore systematically whether higher levels of inequality are related to clientelism, corruption and fraud. Current studies show tentative evidence in favor of this hypothesis (Jong-Sung and Khagram, 2005; Ziblatt, 2009), but more work remains to be done.

Finally, it would be especially interesting to further examine the relative inefficiencies of tax-and-redistribution schemes and clientelistic exchanges. Our theory currently combines two standard assumptions in the literature: tax and redistribution schemes are

\textsuperscript{32} Inequality_{i,t−1} and Inequality^{2}_{i,t−1} just miss statistical significance in model 9, with \( p \)-values of 0.145 and 0.122, respectively.
inefficient while vote buying is a direct exchange between buyers and sellers. Of course, both types of transfers of resources to the poor may increase aggregate income, for example when the poor are credit constrained and under invest in human capital (Saint-Paul and Verdier, 1993; Benabou, 1996). Still, as long as tax and redistribution schemes remain relatively less efficient than clientelistic exchanges the main results of our theory continue to hold. To see this, continue to assume that the rich stand to pay greater net taxes as inequality increases. Then, as inequality increases, it is still the case that the rich pay greater bribes to prevent the election in a greater set of circumstances. In other words, the relative inefficiency of the tax and redistribution scheme means that the rich are willing to offer enough bribes to offset the poor increased preference for redistribution. Given that a large bureaucratic apparatus is needed for a tax and redistribution scheme, the assumption of relative inefficiency appears as an entirely reasonable starting point. However, future empirical research should assess the relative inefficiency costs of these two programs. We leave this question for future research.

APPENDIX

(Proof of Claim 2). The logic follows closely from Dal Bo (2007), Proposition 3, when bribes are contingent on the outcome of the election. Intuitively, the lobby should target a minimum-winning coalition at lowest cost (Claims 2.1, 2.2 and 2.3). We briefly sketch the proof of these steps. Next we note that since there is an aggregate shock on preferences, the lobby faces a risk-return tradeoff in setting its optimal bribes: by increasing its bribes, it can secure the election of the right candidate with greater probability (i.e., for a greater set of preference shocks). Claim 2.4 shows that the lobby would never target a wealthy voter.

Call $M(b(.))$ the set of median voters $m$ for a given bribing schedule $b(.)$, i.e., the left candidate is elected ($E = l$) if and only if $n(m, b(m), s) \geq 0$.

Claim 2.1 Let $P_+(b^*(.)) = \{i \in P | b^*(i) > 0\}$ and $W_+(b^*(.)) = \{i \in W | b^*(i) > 0\}$. $P_+(b^*(.)) \cup W_+(b^*(.)) \subset M(b^*(.))$.

Details omitted (available upon request). If voter $i$ were offered bribe but did not belong to the set of median voters, then the lobby could obtain the same expected utility at strictly lower cost, by reducing the bribe to $i$.³⁴

Claim 2.2 If $P_+(b^*(.)) \neq \emptyset$, then $|P_+(b^*(.))| = \frac{|N|+1}{2} - |W|$ and there exists $b_P$ such that $b^*(i) = b_P$ for any $i \in P_+(b^*(.))$.

³³ More specifically, aggregate income may increase with tax and redistribution (if $n_P^L(\tau^L)|P| + n_W^L(\tau^L)|W| > n_P^L(0)|P| + n_W^L(0)|W|$) and with clientelistic transfers (if $\gamma < 1$). Yet if we assume that clientelism is relatively less inefficient than tax and redistribution schemes, i.e., $\gamma$ is sufficiently small, the logic would go through (see (A.14), taking $\gamma$ to 0).

³⁴ If $\gamma = 1$, then we could have some wealthy voters receiving some positive bribes, if their votes never affected the election outcome (technically, we could have $W_+(b^*(.)) \not\subset M(b^*(.))$). Intuitively, the cost paid by the lobby would be directly offset by the benefit received by the wealthy voter.
By Claim 2.1, if \( P_+(b^*(.)) \neq \emptyset \), then \( |P_+(b^*(.))| \geq \frac{|N|+1}{2} - |W| \) and there exists \( b_p \) such that \( b^*(i) = b_p \) for any \( i \in P_+(b^*(.)) \). Next, it should be clear that we would never have \( |P_+(b^*(.))| > \frac{|N|+1}{2} - |W| \). Details are omitted (available upon request), but let us give some intuition. If \( |P_+(b^*(.))| > \frac{|N|+1}{2} - |W| \), then it is clear that the same election outcomes could be obtained at strictly lower cost for the lobby. Either the lobby could reduce the bribe to a poor voter or it could target a wealthy voter instead of a poor voter.

**Claim 2.3** If \( W_+(b^*(.)) \neq \emptyset \), then \( W_+(b^*(.)) = W \) and there exists \( b_W \) such that \( b^*(i) = b_W \) for any \( i \in W \), where \( b_W \) satisfies

\[
ny^P(\tau^L) - ny^P(0) - b_p = ny^W(\tau^L) - ny^W(0) - b_W
\]

(A.1)

It follows clearly from Claims 2.1 and 2.2. If \( W_+(b^*(.)) \neq \emptyset \), then \( |P_+(b^*(.))| = \frac{|N|+1}{2} - |W| \) so that \( W_+(b^*(.)) = W \). Claim 2.1 implies that bribes \( b_p \) and \( b_W \) must satisfy (A.1).

**Claim 2.4** \( W_+(b^*(.)) = \emptyset \).

Assume not. By Claims 2.1 and 2.2, \( W_+(b^*(.)) = W \subset M(b^*(.)) \). Consider another schedule \( b'(i) \) where \( W_+(b'(.) = \emptyset \), \( |P_+(b'(.)| = \frac{|N|+1}{2} - |W| \) and

\[
ny^P(\tau^L) - ny^P(0) - b'(i'') = ny^W(\tau^L) - ny^W(0)
\]

for any \( i'' \in P_+(b'(.) \). By Claim 2.1, \( W \subset M(b'(.) \).

Now pick \( i' \in W \). The left candidate is elected under bribing schedule \( b^*(.) \) if and only if \( n(i', b^*(i'), s) \geq 0 \Leftrightarrow s \leq \bar{s} \equiv n_d^W(\tau^L) - ny^W(0) - b^*(i') \) and the left candidate is elected under bribing schedule \( b'(.) \) if and only if \( n(i', b'(i'), s) \geq 0 \Leftrightarrow s \leq \bar{s} \equiv n_d^W(\tau^L) - ny^W(0) \). Naturally, \( b^*(i') > 0 \) implies that \( \bar{s} < \bar{s} \). Therefore,

\[
u^{W}(\tau^L, \tau^R, b^*(.)) - u^{W}(\tau^L, \tau^R, b'(.) = \int_{\bar{s}}^{\bar{s}} \left( \sum_{i \in W} n_d^W(0) + s - (\gamma - 1)b^*(i) - ny^W(\tau^L) - \gamma \sum_{i \in P} b^*(i) \right) dF(s) - \int_{\bar{s}}^{\infty} \left( (\gamma - 1) \sum_{i \in W} b^*(i) + \gamma \sum_{i \in P} (b^*(i) - b'(i)) \right) dF(s)\]

so that

\[
u^{W}(\tau^L, \tau^R, b^*(.)) - u^{W}(\tau^L, \tau^R, b'(.) < \sum_{i \in W} \left[ \int_{\bar{s}}^{\bar{s}} ny^W(0) + s - ny^W(\tau^L) dF(s) \right] < 0
\]

The first inequality follows from \( \sum_{i \in P} b^*(i) > \sum_{i \in P} b'(i) > 0 \), \( \sum_{i \in W} b^*(i) > 0 \) and \( \gamma > 1 \) and the second follows from the definition of \( \bar{s} \). \( \square \)
(Proof of Proposition 1). The problem for the lobby can be written as

\[
\max_{b_p} v(b_p; \theta, y) = -\left[ \frac{|N| + 1}{2} - |W| \right] \gamma b_p (1 - F(\tau)) \\
+ |W| \left[ \int_{-\infty}^{T} n y^W (\tau^L) dF(s) + \int_{\tau}^{\infty} (n y^W (0) + s) dF(s) \right]
\]

such that

\[
0 \leq b_p \leq [n y^P (\tau^L) - n y^P (0)] + [n y^W (0) - n y^W (\tau^L)]
\]

where

\[
\tau = n y^P (\tau^L) - n y^P (0) - b_p
\]

We have

\[
\frac{\partial v(b_p; \theta, y)}{\partial b_p} = -\left[ \frac{|N| + 1}{2} - |W| \right] \gamma (1 - F(\tau)) \\
+ f(\tau) \left[ -\left[ \frac{|N| + 1}{2} - |W| \right] \gamma b_p + |W|[n y^W (0) - n y^W (\tau^L) + \tau] \right]
\]

(A.5)

Note from (A.5) that \(\frac{\partial v(b_p; \theta, y)}{\partial b_p} < 0\) for any \(b_p \geq \bar{b}_p\), where

\[
\bar{b}_p = \left( \frac{|W|}{\left( \frac{|N| + 1}{2} - |W| \right) \gamma + |W|} \right) \left[ [n y^P (\tau^L) - n y^P (0)] + [n y^W (0) - n y^W (\tau^L)] \right]
\]

so that the optimal level of bribes \(b^*_p\) is such that \(b^*_p \in [0, \bar{b}_p]\).

If the solution is such that \(b^*_p > 0\), it must satisfy the first-order condition \(\frac{\partial v(b^*_p; \theta, y)}{\partial b_p} = 0\) and the second-order condition \(\frac{\partial^2 v(b^*_p; \theta, y)}{\partial b_p^2} < 0\). Let us prove that the second-order condition holds whenever the first-order condition holds. This will imply that if the solution is such that \(b^*_p > 0\), there is a unique value \(b_p\) which satisfies the first-order condition.

We have

\[
\frac{\partial^2 v(b_p; \theta, y)}{\partial b_p^2} = -\left[ \frac{|N| + 1}{2} - |W| \right] \gamma f(\tau) \\
- f(\tau) \left[ \left[ \frac{|N| + 1}{2} - |W| \right] \gamma b_p + |W|[n y^W (0) - n y^W (\tau^L) + \tau] \right]
\]

(A.6)
Also, note that Condition 1 implies

$$\frac{\partial f(s)}{\partial s} = \frac{f'(s)[1 - F(s)] + f(s)^2}{[1 - F(s)]^2} > 0$$

$$\Rightarrow -f'(s) \frac{[1 - F(s)]}{f(s)} < f(s)$$  \hspace{1cm} (A.7)

Therefore, take a point $b'_p$ such that $\frac{\partial v(b'_p; \theta, y)}{\partial b_p} = 0$. Replacing in (A.6), we have

$$\frac{\partial^2 v(b'_p; \theta, y)}{\partial b_p^2} < -f(\bar{b}) \left[ \left( \frac{|N| + 1}{2} - |W| \right) \gamma + |W| \right] < 0$$  \hspace{1cm} (A.8)

Therefore, if there is a value $b_p^* = \frac{|W|}{|N|}$ such that $b_p^* = 0$ if and only if $\theta \in \left( \frac{|W|}{|N|}, \bar{\theta} \right]$.

We want to characterize cases where $b_p^*$ is strictly positive. We show that there exists a value $\bar{\theta} \in \left( \frac{|W|}{|N|}, 1 \right]$ such that $b_p^* = 0$ if and only if $\theta \in \left( \frac{|W|}{|N|}, \bar{\theta} \right]$.

First,

$$\frac{\partial v(b_p; \frac{|W|}{|N|}, y)}{\partial b_p} = - \left( \frac{|N| + 1}{2} - |W| \right) \gamma (1 - F(-b_p))$$

$$-f(-b_p) b_p \left[ \left( \frac{|N| + 1}{2} - |W| \right) \gamma + |W| \right]$$

$$< 0$$

so that, by continuity, there exists $\bar{\theta} > \frac{|W|}{|N|}$ such that $b_p^* = 0$ if $\theta \in \left( \frac{|W|}{|N|}, \bar{\theta} \right]$.

Second, assume that $b_p^*$ is interior ($b'_p \in [0, \tilde{b}_p]$), then we have $\frac{\partial b_p^*}{\partial \theta} > 0$. To see this, use the implicit function theorem on $\frac{\partial v(b_p^*, \theta, y)}{\partial b_p} = 0$, so that

$$\frac{\partial b_p^*}{\partial \theta} = -\frac{\frac{\partial^2 v(b_p^*, \theta, y)}{\partial b_p^2 \partial \theta}}{\frac{\partial^2 v(b_p^*, \theta, y)}{\partial b_p^2}}$$  \hspace{1cm} (A.9)

We know that $\frac{\partial^2 v(b_p^*, \theta, y)}{\partial b_p^2} < 0$. We have

$$\frac{\partial^2 v(b_p^*, \theta, y)}{\partial b_p^2 \partial \theta} = \frac{d(ny^P(\tau_L) - ny^P(0))}{d \theta} f(\bar{b}) \left( \left( \frac{|N| + 1}{2} - |W| \right) \gamma + |W| \right)$$

$$+ \frac{d(ny^W(0) - ny^W(\tau_L))}{d \theta} f(\bar{b}) |W|$$
\[ + \frac{d(ny^P(\tau_L) - ny^P(0))}{d\theta} \left[ - \left( \frac{|N|}{2} - |W| \right) \gamma b_P \right] \]

\[ + |W| \left[ ny^W(0) - ny^W(\tau_L) + \frac{3}{2} \right] \]

(A.10)

which reduces to the following, using the first-order condition

\[ \frac{\partial^2 v(b^*_P, \theta, y)}{\partial b_P \partial \theta} = \frac{d(ny^P(\tau_L) - ny^P(0))}{d\theta} \]

\[ \times \left[ \left( \frac{|N|}{2} - |W| \right) \gamma \left( f(\bar{s}) + \frac{1 - F(\bar{s})}{f(\bar{s})} f'(\bar{s}) \right) + f(\bar{s}) |W| \right] \]

\[ + \frac{d(ny^W(0) - ny^W(\tau_L))}{d\theta} f(\bar{s}) |W| \]

(A.11)

We then note, using the envelope theorem,

\[ \frac{d(ny^P(\tau_L) - ny^P(0))}{d\theta} = \frac{d}{d\theta} \left( (1 - \tau_L) y^P + (\tau_L - C(\tau_L)) \frac{y}{|N|} - y^P \right) \]

\[ = \left[ -y^P + (1 - C'(\tau_L)) \frac{y}{|N|} \right] \frac{\partial \tau_L}{\partial \theta} - \tau_L \frac{\partial y^P}{\partial \theta} \]

\[ = -\tau_L \frac{\partial y^P}{\partial \theta} > 0 \]  

(A.12)

\[ \frac{d(ny^W(0) - ny^W(\tau_L))}{d\theta} = \frac{d}{d\theta} \left( y^W - ((1 - \tau_L) y^W + (\tau_L - C(\tau_L)) \frac{y}{|N|}) \right) \]

\[ = \left[ y^W - (1 - C'(\tau_L)) \frac{y}{|N|} \right] \frac{\partial \tau_L}{\partial \theta} + \tau_L \frac{\partial y^W}{\partial \theta} \]

\[ = \left[ y^W - C'(\tau_L) \frac{y}{|N|} \right] \frac{\partial \tau_L}{\partial \theta} + \tau_L \frac{\partial y^W}{\partial \theta} > 0 \]  

(A.13)

so that, using (A.7),

\[ \frac{\partial^2 v(b^*_P, \theta, y)}{\partial b_P \partial \theta} > 0. \]

Finally, let us establish that there exists \( \bar{y} \) such that for any \( y > \bar{y}, \bar{\theta} < 1 \). Note that

\[ \frac{\partial v(0, 1, y)}{\partial b_P} = - \left[ \frac{|N|}{2} - |W| \right] \gamma \left[ 1 - F \left( \frac{(\tau_L - C(\tau_L)) y}{|N|} \right) \right] \]

\[ + f \left( (\tau_L - C(\tau_L)) \frac{y}{|N|} \right) \tau_L y \]
so that \( \frac{\partial v(\theta, y)}{\partial b_P} > 0 \) if and only if

\[
\frac{f \left( (\tau^L - C(\tau^L)) \frac{\gamma}{N^T} \right)}{1 - F \left( (\tau^L - C(\tau^L)) \frac{\gamma}{N^T} \right)} > \frac{\left\lceil \frac{|N| + 1}{2} - |W| \right\rceil}{\tau^L y}
\]

Therefore, by Condition (1), the left-hand side is increasing in \( y \), while, clearly, the right-hand side is decreasing in \( y \). The conclusion follows. \( \blacksquare \)

(Proof of Corollary 1). \( \text{prob}(E = L) = F(ny^P(\tau^L) - ny^P(0) - b^*_p) \) so that

\[
\frac{\partial \text{prob}(E = L)}{\partial \theta} = f(ny^P(\tau^L) - ny^P(0) - b^*_p) \left[ \frac{d(ny^P(\tau^L) - ny^P(0))}{d\theta} - \frac{\partial b^*_p}{\partial \theta} \right]
\]

Since \( f(ny^P(\tau^L) - ny^P(0) - b^*_p) > 0 \), \( \frac{\partial \text{prob}(E = L)}{\partial \theta} \) takes the same sign as \( \frac{d(ny^P(\tau^L) - ny^P(0))}{d\theta} - \frac{\partial b^*_p}{\partial \theta} \).

For any \( \theta < \bar{\theta}, b^*_p = 0 \). Therefore, \( \frac{\partial \text{prob}(E = L)}{\partial \theta} |_{\theta < \bar{\theta}} > 0 \) since, by (A.12), \( \frac{d(ny^P(\tau^L) - ny^P(0))}{d\theta} > 0 \).

For any \( \theta > \bar{\theta}, b^*_p > 0 \) and is given by \( \frac{\partial v(b^*_p, \theta, y)}{\partial b_P} = 0 \). Therefore, using Equations (A.5), (A.6), (A.9), and (A.10), we get

\[
\frac{d(ny^P(\tau^L) - ny^P(0))}{d\theta} - \frac{\partial b^*_p}{\partial \theta} \bigg|_{\theta > \bar{\theta}} = -f(s) \left[ \frac{d(ny^W(\tau^L) - ny^W(0))}{d\theta} \left\lceil \frac{|N| + 1}{2} - |W| \right\rceil \gamma - \frac{d(ny^W(0) - ny^W(\tau^L))}{d\theta} |W| \right]
\]

\[
= -yC'(\tau^L) \frac{\partial \tau^L}{\partial \theta}
\]

If \( \left\lceil \frac{|N| + 1}{2} - |W| \right\rceil \gamma < |P| \), then

\[
\frac{d(ny^P(\tau^L) - ny^P(0))}{d\theta} \frac{d(ny^W(0) - ny^W(\tau^L))}{d\theta} |W| < 0
\]
Since \( \frac{\partial^2 v(b^*, \theta, y)}{\partial b^2} < 0 \), we have

\[
\left. \frac{\partial \text{prob}(E = L)}{\partial \theta} \right|_{\theta = \bar{\theta}} < 0
\]

REFERENCES


